Grade 8 Mathematics
Number and Number Relations: Lesson 14

Read aloud to the students the material that is printed in **boldface type** inside the boxes. Information in regular type inside the boxes and all information outside the boxes should **not** be read to students. Possible student responses are included in parentheses after the questions.

NOTE: The directions read to students may depend on the available materials. Read only those parts of the lesson that apply to the materials you are using.

Any directions that ask you to do something, such as to turn to a page or to hand out materials to students, will have an arrow symbol ( ) by them.

Purpose of Lesson 14:

- In this lesson, the tutor and the students will
  - ✓ demonstrate a conceptual understanding of proportional reasoning,
  - ✓ determine equivalent ratios, and
  - ✓ find the missing amount in a proportion.

Equipment/Materials Needed:

- Copies of Student Sheets 88 and 89
- Paper and pencils
- Chalkboard

Preparations before beginning Lesson 14:

- Run one copy of Student Sheets 88 and 89 for each student.
- Have paper and pencils available.

Lesson 14: Number and Number Relations

Say:

In this lesson, you will compare quantities. There are two ways to compare quantities. You can use subtraction or you can use a ratio. Let's look at a problem. August and June each have collections of baseball cards. August has 60 cards and June has 90 cards. How much bigger is June's collection than August's?

Subtraction: **90 cards – 60 cards = 30 cards.**

June has 30 cards more than August.

Ratio: $\frac{90 cards}{60 cards} = \frac{9 cards}{6 cards} = \frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$.

June's collection is $1\frac{1}{2}$ times larger than August's.

Say:

***Ratios* compare two numbers. Suppose you have seven kittens and three are males. You can compare a part of something to the whole or all of it.**

3 male kittens to 7 kittens in all or $\frac{3}{7}$

You can compare a part of something to another part of it.

3 male kittens to 4 female kittens or $\frac{3}{4}$

Write this problem on the board. There are 15 girls and 20 boys in the class.

Say:

If you compare the number of girls to the number of boys, you could write the ratio is the following ways:

15 girls to 20 boys or 15 to 20

15 girls : 20 boys or 15 : 20

$\frac{15\ girls}{20\ boys}$ **or** $\frac{15}{20}$

All of these forms represent a comparison of 15 girls to 20 boys. If you use the fractional form, you can write the fraction is lowest terms.

$\frac{15\ girls}{20\ boys}$ **or** $\frac{15}{20}$ **or** $\frac{3\ girls}{4\ boys}$ **or** $\frac{3}{4}$

Give students Student Sheet 88.

Answers:

1. 4 to 3, 4 : 3, or $\frac{4}{3}$
2. 3 to 4, 3 : 4, or $\frac{3}{4}$
3. 4 to 12, 4 : 12, or $\frac{4}{12}$ or $\frac{1}{3}$
4. 3 to 5, 3 : 5, or $\frac{3}{5}$
5. 11 to 1, 11 : 1, or $\frac{11}{1}$
6. 174 to 157
7. 131 to 174
8. 305 to 319
9. 157 to 174
10. 157 to 624

Say:

Equivalent fractions name the same amount. *Equivalent ratios* also name the same amount. Let's look at this problem. I have five fingers on one hand. The ratio "1 hand to 5 fingers" can be used to represent this amount. I have 10 fingers on two hands. The ratio "2 hands to 10 fingers" can be used to represent this amount. Are these two ratios equivalent? Does $\frac{1}{5} = \frac{2}{10}$? (Yes.) How did you determine that the two ratios were equivalent? Answers may vary, but here are some samples.

(To check to see whether two ratios are equivalent, you could do any of the following:

a. Use a calculator or your head to write both ratios as decimals and to compare them. 0.2 = 0.2.

b. Find a common denominator and compare the two fractions. The common denominator is 10. 1/5 is the same as 2/10 and 2/10 = 2/10.

c. Think of equivalent fractions. I can multiply both the numerator and the denominator in 1/5 by 2. If I do, I get 2/10, so 2/10 = 2/10.)

Write this ratio on the board. $\frac{10}{15}$

Say:

Find a ratio that is equivalent to the ratio, $\frac{10}{15}$. (2/3, 20/30, etc.) **How did you find the equivalent ratio.** Answers will vary, but here are some samples. (I divided both the numerator and denominator by five. This operation gave me 2/3. I multiplied both the numerator and denominator by two. This operation gave me 20/30.)

Find three ratios that are equivalent to 4/8. (1/2, 2/4, 3/6, 12/24, etc.)

Say:

A *proportion* is a statement that shows two ratios are equivalent. In the problems above, $\frac{10}{15} = \frac{2}{3}$ and $\frac{10}{15} = \frac{20}{30}$ are proportions. Proportions are types of equations; so just like equations, some proportions are true and some are false. If you are asked to determine whether a proportion is true, you simply find out whether the two ratios are equivalent.

Draw this table on the board.

Hands	1	2
Fingers	5	10

Say:

You could write several proportions from this table.

$$\frac{1\ hand}{5\ fingers} = \frac{2\ hands}{10\ fingers} \qquad \frac{5\ fingers}{1\ hand} = \frac{10\ fingers}{2\ hands}$$

$$\frac{5\ fingers}{10\ fingers} = \frac{1\ hand}{2\ hands} \qquad \frac{10\ fingers}{5\ fingers} = \frac{2\ hands}{1\ hand}$$

You must be careful. The units of the ratios must match. It is wise to write the units in the ratio first and then rewrite the ratio without the units.

$$\frac{1\ hand}{5\ fingers} \quad 1 \quad \frac{10\ fingers}{2\ hands}$$

Say:

Sometimes a proportion will be missing a term. You know that, if two ratios name a proportion, the ratios must be equivalent. You can use this idea to solve for the missing term. Let's look at this problem. George uses a mixture of two gallons of red paint to three gallons of white paint to make the pink color that his customer wants. How many gallons of paint does this combination make? (5 gallons) **The customer needs 15 gallons of paint. How many gallons of white paint will George need to make the 15 gallons?** (9 gallons) **How did you find this amount?**
Answers will vary, but here are some possible ways.

Think: 3 gallons of white to get 5 gallons of paint
6 gallons of white to get 10 gallons of paint
9 gallons of white to get 15 gallons of paint.

Set up a proportion:

$$\frac{\text{3 gallons of white}}{\text{5 gallons of paint}} = \frac{x \text{ gallons of white}}{\text{15 gallons of paint}}$$

$$\frac{3}{5} = \frac{x}{15}$$

To get to 15 from 5, I multiply by 3. I need to multiply the denominator by the same amount, so 3 times 3 is 9. George needs 9 gallons of white paint.

- Give students Student Sheet 89.

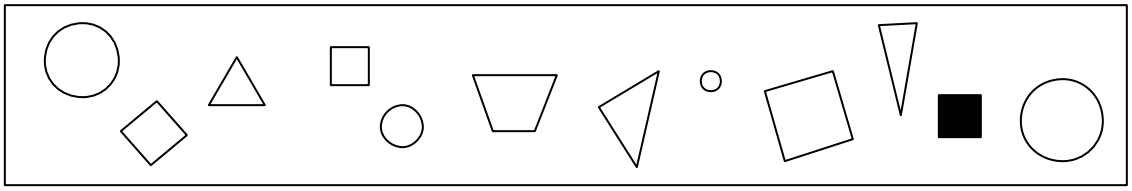
Answers:

1. C	2. A, B	3. A
4. D	5. 12 pens	6. $0.75
7. 72 inches	8. 90 minutes	9. 48 ounces

- Have one student summarize today's lesson. Proportional reasoning is an extremely important concept. Students will see this concept again in measurement and geometry.

Student Sheet 88 (Number Relations: Lesson 14)

Using the figures in the box, write ratios for problems 1 – 5. Write the ratios in three different forms, using the word "to," using a colon, and using a fraction.



1. the ratio of the number of circles to the number of triangles

2. the ratio of the number of triangles to the number of circles

3. the ratio of the number of squares to the total number of figures

4. the ratio of the number of three-sided figures to the number of quadrilaterals

5. the ratio of the number of white figures to the number of black figures

At Riverdale Junior High School, there are 305 students in the 7th grade, 174 of whom are girls. In the 8th grade class, there are 162 boys and 157 girls. Use this information to write ratios for problems 6 – 10. Choose the ratio form that you prefer.

6. the ratio of the number of 7th grade girls to the number of 8th grade girls

7. the ratio of the number of 7th grade boys to the number of 7th grade girls

8. the ratio of the number of 7th grade students to the number of 8th grade students

9. the ratio of the number of 8th grade girls to the number of 7th grade girls

10. the ratio of the number of 8th grade girls to the total number of students in both grades

Student Sheet 89 (Number Relations: Lesson 14)

1. All of the following ratios, except one, represent the same amount. Which ratio does not belong?

   A. 4 : 16 B. 1 : 4 C. 16 to 4 D. $\frac{4}{16}$

2. Which proportions are true?

   A. $\frac{3}{6} = \frac{18}{36}$ B. $\frac{1}{5} = \frac{3}{15}$ C. $\frac{2}{4} = \frac{4}{16}$ D. $\frac{2}{3} = \frac{12}{6}$

3. On their summer vacation, Bobbie's family drove 200 miles in four hours. Which proportion shows about how long it should take them to drive 650 miles?

   A. $\frac{200}{4} = \frac{650}{x}$ B. $\frac{650}{200} = \frac{4}{x}$ C. $\frac{650}{4} = \frac{200}{x}$ D. $\frac{4}{200} = \frac{650}{x}$

4. Blank video tapes can be purchased at a price of four for $19. Which proportion shows how many tapes can be bought for $114?

   A. $\frac{4}{19} = \frac{114}{x}$ B. $\frac{114}{19} = \frac{4}{x}$ C. $\frac{114}{4} = \frac{19}{x}$ D. $\frac{4}{19} = \frac{x}{114}$

5. Ballpoint pens are priced two for 89 cents. How many pens can be bought for $5.34?

6. Annie wanted to buy three oranges. If the clerk said oranges were priced six for $1.50, how much would three oranges cost?

7. There are 12 inches in one foot. How many inches are in six feet?

8. Benny determines that he can review four sections of his geography chapter in 40 minutes. How long should it take him to review nine sections, if he continues at this rate?

9. There are 32 ounces in two pounds. How many ounces are in three pounds?