

## Grade 8 Mathematics

### Number and Number Relations: Lesson 4

Read aloud to the students the material that is printed in **boldface type** inside the boxes. Information in regular type inside the boxes and all information outside the boxes should **not** be read to students. Possible student responses are included in parentheses after the questions.

NOTE: The directions read to students may depend on the available materials. Read only those parts of the lesson that apply to the materials you are using.

Any directions that ask you to do something, such as to turn to a page or to hand out materials to students, will have an arrow symbol ( $\Rightarrow$ ) by them.

#### *Purpose of Lesson 4:*

- In this lesson, the tutor and the students will
  - ✓ estimate whole number and decimal quantities, and
  - ✓ determine the reasonableness of results in real-world problems.

#### Equipment/Materials Needed:

- Copies of Student Sheets 9 and 10
- Paper and pencils.

#### *Preparations before beginning Lesson 4:*

- Run off one copy of Student Sheets 9 and 10 for each student.  
Cut Student Sheet 9 into 2 parts.
- Have paper and pencils available.

## Lesson 4: Number and Number Relations

Say:

**This lesson will focus on estimation and reasonable results. There are many ways to estimate, more than just rounding numbers. If you learn how to estimate well, you can really improve your scores on multiple choice tests, because you will learn to eliminate many of the wrong answers. Since we are not looking for the exact answer, often we will have different estimates. As long as the estimates are fairly close, arriving at different estimates is really okay.**

⇒ Read this problem to the students. Play tickets were sold for 3 days last week. The sales for the 3 days were 568 tickets, 412 tickets, and 392 tickets. About how many tickets were sold on the 3 days?

Write the following problem on the board.

$$\begin{array}{r} 568 \\ 412 \\ + 392 \\ \hline \end{array}$$

Say:

**I want you to estimate the answer to the problem. In a minute, we will talk about the different methods you used to estimate. Allow students time to estimate the answer.**

Say:

**One of the easiest ways to estimate is to use *front-end estimation*. This estimation procedure is easiest to do with addition, but can be used with subtraction and multiplication. We will use only the largest, or leading, digit in each number and forget about the others for now.**

⇒ As you work through the problem, write your steps on the board.

$$\begin{array}{r} 568 \quad \text{think 5 or } 500 \\ 412 \quad \text{think 4 or } 400 \\ + 392 \quad \text{think 3 or } 300 \\ \hline \end{array}$$

Say:

**We have  $5 + 4 + 3$ , or 12; but this total is 12 hundreds or 1200. This number is an estimate of the sum, but not a very good one. Whenever you use front-end estimation, you must adjust your estimate.**

**So far, you have used only the hundreds place. Look at the next digit in each number. What place is this? (tens) About how many tens do you think we should add to the 1200? (16 or 17 tens) If you add 160 or 170 to 1200, you get either 1360 or 1370. Either number is a good estimate of the problem. Did anyone estimate this way? (If any of the students did use this method, allow them to talk about how they estimated.) Remember that, when using front-end estimation, you must adjust your first estimate.**

Say:

**Another method for rounding is to use *compatible numbers*. *Compatible numbers* are numbers that are easy to work with. They are also called *nice numbers*. Some examples are  $25 + 75$ ,  $80 - 20$ ,  $4 \times 25$ , and  $250 \div 50$ . When we use this method in estimating, we make the numbers in the problem more compatible or easier to work with. For the problem above, do you see any numbers that you could make easier to add? (568 is about 600 and 412 is about 400. **What would that sum be? (1000) How should you finish the problem? (I add about 400 more. I get 1400 as my estimate.) Did anyone estimate this way? (If any of the students did use this method, allow them to talk about how they estimated.) There are different ways to make numbers compatible, so again, the estimates may be a little different.****

Say:

**Another method of estimating is to *round* numbers. When we round numbers, we have to know to which place value we want to round. Let's look back at the same problem. I want you to round each number to the nearest hundred. Let's look at 568. The place to which we are rounding is the hundreds, so to which digit should we look? (tens) **What is the rule for rounding? (If this digit is 5 or above, round up. If this digit is less than 5, round down.) So to what would 568 round? (600)****

⇒ As you work through the problem, write your steps on the board.

568	rounds to	600
412	rounds to	400
+ 392	rounds to	400
-----		-----

Say:

**To what number would the other numbers round?** (412 rounds to 400, because one (1) ten is less than 5 tens, so you round down. 392 rounds to 400 because 9 tens is greater than 5 tens, so you round up.) **What numbers do you add?** ( $600 + 400 + 400$ .) **What is your estimate?** (1400.) **Did anyone estimate this way?** (If any of the students did use this method, allow them to talk about how they estimated.)

Say:

**We could also round these numbers to the nearest tens. If we round to the nearest ten, to which digit should we look?** (ones) **What would each number round to?** (568 would round to 570, because 8 ones is greater than 5 ones, so you round up. 412 would round to 410, because 2 ones is less than 5 ones, so you round down. 392 rounds to 390, because 2 ones is less than 5 ones.) **We add  $570 + 410 + 390$  or  $1370$ .** **Did anyone estimate this way?** (If any of the students did use this method, allow them to talk about how they estimated.)

⇒ Give the top part of Student Sheet 9 to the students.

Say:

**Let's look back at the problem that we just completed and think about each method. Did the methods all give estimates that were fairly close?** (Yes.) **Which method did you like best?** (Answers will vary. Usually if you can make compatible numbers, this method is the easiest way to estimate. Also front-end estimation is quick, but you have to adjust. Rounding to the smaller place values, such as the tens in this problem, is usually the most accurate; but this method takes the most time.)

Say:

**I am going to read another problem. Cassie went shopping for presents. She spent \$24.56 on a video, \$8.92 on a book, and \$75.20 on PlayStation games. About how much did she spend?**

⇒ Ask one of the students to write the problem on the board. This procedure will help reinforce lining up decimals in addition.

$$\begin{array}{r} \$ 24.56 \\ \phantom{\$} 8.92 \\ + \phantom{\$} 75.20 \\ \hline \end{array}$$

Say:

**I want you to try all 4 methods of estimating: front-end estimation, making compatible numbers, rounding to the nearest \$10, and rounding to the nearest \$1.** Allow time for students to complete this activity.

⇒ Give the students the bottom half of Sheet 9. This sheet summarizes each method.

Say:

**Let's look at each method.** Allow students time to discuss each method. **Which method did you like best? Why?** (Answers will vary.)

Say:

**Let's look at the same estimating methods for subtraction. I want to buy a printer that costs \$845. 20. I have only \$667.85 saved. About how much more money do I need to buy the printer?**

⇒ Have a student write the problem on the board.

$$\begin{array}{r} 845.20 \\ - 667.85 \\ \hline \end{array}$$

Say:

**Let's look at front-end estimation. What numbers would I use to subtract?** (800 – 600.) **What is the estimate?** (\$200) **How would I adjust this estimate?** (I might think that I need \$45 which is about \$50, but I have about \$70, so I can subtract \$20 from my estimate.) **What is the estimate?** (\$180)

Say:

**Do you see any compatible numbers in this problem?** (They probably will have trouble finding any. Compatible numbers do not really fit this problem. You could try 840 minus 670 or something similar, but it is not really effective.) Students need to learn that the most efficient estimation strategy varies for different problems.

Say:

**Let's try rounding to the nearest hundred dollars. What numbers would you use to subtract?** (800 – 700.) **What is the estimate?** (\$100)

**What if you round to the nearest ten dollars? What numbers would you use to subtract? (850 – 670.) What is the estimate? (\$180) Should you try rounding to the nearest \$1? (You could, but you no longer will get a quick estimate.) What would you subtract? (845 – 668.) What is the estimate? (\$177)**

⇒ Give the following problems to the students to estimate. Write them on the board one at a time. Allow time for discussion.

- |                        |                        |
|------------------------|------------------------|
| 1) $3222 + 825 + 6506$ | 2) $3.89 + 5.3 + 6.12$ |
| 3) $829 - 498$         | 4) $24.65 - 12.95$     |

Answers:

- |                           |             |
|---------------------------|-------------|
| 1) About 10,400 to 11,000 | 2) About 15 |
| 3) About 300 to 350       | 4) About 12 |

Say:

**Let's look at the estimation methods for multiplication. The school needs to purchase 297 calculators. They calculators cost \$4 each. About how much will the calculators cost?**

⇒ Have a student write the problem on the board.

$$\begin{array}{r} 297 \\ \times 4 \\ \hline \end{array}$$

Say:

**In multiplication, often we want a *ballpark figure*, one that tells us whether we are close. If we use front-end estimation, what two numbers would we use to multiply? (200 and 4.) What would this give us? (\$800) Do you think that estimate is close? (No.) How should we adjust the estimate? (You might think, 97 is about 100, so you need to multiply 100 times 4. You need to add \$400 to your original estimate, so the estimate is about \$1200.)**

**Do you see any compatible numbers that could help you estimate this problem? (You could multiply  $300 \times 4$  and get \$1200.)**

**If you round the top number to the nearest hundred, what numbers would you use? ( $300 \times 4$ .) What is your estimate? (\$1200.) If you round the top number to the nearest 10, what numbers would you use? ( $290 \times 4$ .) What is your estimate? (\$1160)**

Say:

**Let's look at another multiplication problem. The store ordered 344 packs of 2-color counters. There are about 62 in each pack. The owners want to have at least 20,000 in stock. Did the owners order enough 2-color counters?**

⇒ Have a student write the problem on the board.

$$\begin{array}{r} 344 \\ \times 62 \\ \hline \end{array}$$

Say:

**There are many ways to approach this problem. What do you think we should do?** Let the students work on the problem for a few minutes and discuss what they have done. Here are some samples of possible thinking. (I thought  $300 \times 60$  and that gave me 18,000; but I didn't use 45. Then I thought 45 is about 40, so I multiplied  $40 \times 60$  and got 2,400. When I added 2,400 to 18,000, I got 20,400; or I thought  $300 \times 60 + 40 \times 60$ . This method gave me  $18,000 + 2,400$  or 20,400; or I thought  $350 \times 60$  or  $300 \times 60 + 50 \times 60$  which gave me  $18,000 + 3,000$  or 21,000. In all of the methods, we found that the owners had bought enough.)

⇒ Write the following 3 problems and answers on the board.

1. $\begin{array}{r} 46.8 \\ \times 5 \\ \hline \end{array}$	2. $\begin{array}{r} 1465 \\ \times 36 \\ \hline \end{array}$	3. $\begin{array}{r} 987 \\ \times 18 \\ \hline \end{array}$
2340	52740	177666

Say:

**Two of the 3 problems on the board have been worked incorrectly. Use estimation to tell me which two are wrong. Be ready to explain how you determined the answers were wrong.** (Numbers 1 and 3 are incorrect. In Number 1, if you think  $50 \times 5$ , the answer is about 250. In Number 3, if you think  $1000 \times 20$ , the answer is about 20,000.)

Say:

**Any of the above methods can help you decide whether an answer is reasonable. It is usually best to use the quickest method first. If that method does not eliminate all of the wrong answers, try another method.**

⇒ Give Student Sheet 10 to the students. Have the students work the problems one at a time. As they finish a problem, discuss the methods they used to find the most reasonable estimate.

Answers:

1) D      2) D      3) B      4) C      5) B      6) D

7) C We have not really touched on division in this lesson. If faced with a problem like this one, the students can think multiplication. Which number times 40 will give me approximately 300 or 320?

8) D

⇒ Have one student summarize today's lesson. The students should understand that there are many ways to estimate problems and that different methods work better for different problems.

## Student Sheet 9 (Number: Lesson 4)

### Estimation Methods in Addition

#### Front End Estimation

$$\begin{array}{r} 568 \\ 412 \\ + 392 \\ \hline \end{array} \qquad \begin{array}{r} 500 \\ 400 \\ + 300 \\ \hline 1200 \end{array}$$

I have to adjust by 16 or 17 tens, so I get 1360 or 1370.

#### Rounding to the Nearest Hundred

$$\begin{array}{r} 568 \\ 412 \\ + 392 \\ \hline \end{array} \qquad \begin{array}{r} 600 \\ 400 \\ + 400 \\ \hline 1400 \end{array}$$

The actual answer is 1372.

#### Compatible Numbers

I think: 568 is about 600 and 412 is about 400. Adding 600 and 400 gives me 1000. 392 is about 400, so I add 400 more. I get 1400 as my estimate.

#### Rounding to the Nearest Ten

$$\begin{array}{r} 568 \\ 412 \\ + 392 \\ \hline \end{array} \qquad \begin{array}{r} 570 \\ 410 \\ + 390 \\ \hline 1370 \end{array}$$

---

---

#### Front End Estimation

$$\begin{array}{r} \$ 24.56 \\ 8.92 \\ + 75.20 \\ \hline \end{array} \qquad \begin{array}{r} 20 \\ 8 \\ + 70 \\ \hline 98 \end{array}$$

I have to adjust by \$9 or \$10, so I get \$107 or \$108.

#### Rounding to the Nearest \$10

$$\begin{array}{r} \$ 24.56 \\ 8.92 \\ + 75.20 \\ \hline \end{array} \qquad \begin{array}{r} 20 \\ 10 \\ + 80 \\ \hline \$ 110 \end{array}$$

The actual answer is \$108.68.

#### Compatible Numbers

I think: 24.56 is about 25, and 75.20 is about 75. I add those numbers and get 100. 8.92 is about 9, so I get \$109.

#### Rounding to the Nearest \$1

$$\begin{array}{r} \$ 24.56 \\ 8.92 \\ + 75.20 \\ \hline \end{array} \qquad \begin{array}{r} 25 \\ 9 \\ + 75 \\ \hline \$ 109 \end{array}$$

## Student Sheet 10 (Number: Lesson 4)

**Choose the most reasonable answer without working the problem.**

**Estimate. Be ready to discuss the processes you used.**

1. If a roast costs \$3.89 a pound, which is the best estimate of the cost of a 5 pound roast?  
A. \$4                      B. \$8                      C. \$15                      D. \$20
2. The class treasurer deposited checks of \$49.82, \$8.98, \$50.01, and \$2.45 into the bank account. Which is the best estimate of the amount of the deposit?  
A. \$200                      B. \$182                      C. \$150                      D. \$110
3. Jimmy paid \$1.89 a gallon for gas. Which is a reasonable estimate of the amount that 15 gallons of gas would cost?  
A. \$17                      B. \$30                      C. \$50                      D. \$300
4. Videotapes were advertised at 3 for \$18.95. Connie wanted to buy only 2 tapes. Which would be a reasonable estimate of the cost of 2 tapes?  
A. \$21                      B. \$19                      C. \$13                      D. \$6
5. When Byron left his home for his cross-country trip, the odometer on his car read 2757.6 miles. So far, he has driven 654 miles. Which could be the odometer reading now?  
A. 2823.0 miles                      B. 3411.6 miles  
C. 9297.6 miles                      D. 18034.7 miles
6. Norman Gallagher grew a pumpkin that weighed 612 pounds. It holds the world's record as the largest pumpkin ever recorded. Which is the best estimate of the number of ounces that the pumpkin weighed? (1lb = 16 oz)  
A. 40 oz                      B. 80 oz                      C. 7000 oz                      D. 10,000 oz
7. The 8<sup>th</sup> grade class is going on a field trip. There are 328 students in the class. Which is the best estimate of the number of buses that will be needed if no more than 44 students can ride on the bus?  
A. 370 buses                      B. 190 buses                      C. 8 buses                      D. 4 buses
8. If Georgia pays \$2.27 cents for a burger and 75¢ for a soda, approximately how much did her meal cost?  
A. \$77                      B. \$10                      C. \$8                      D. \$3