Grade 8 Mathematics
Number and Number Relations: Lesson 1

Read aloud to the students the material that is printed in **boldface type** inside the boxes. Information in regular type inside the boxes and all information outside the boxes should **not** be read to students. Possible student responses are included in parentheses after the questions.

NOTE: The directions read to students may depend on the available materials. Read only those parts of the lesson that apply to the materials you are using.

Any directions that ask you to do something, such as to turn to a page or to hand out materials to students, will have an arrow symbol (⇨) by them.

Purpose of Lesson 1:

- In this lesson, the tutor and the students will
  - ✓ recognize fractional parts of a region or a set,
  - ✓ understand the relationship between mixed numbers and improper fractions,
  - ✓ write fractions in simplest form, and
  - ✓ compare fractions.

Equipment/Materials Needed:

- Copies of Student Sheets 1 and 2
- Crayons – 1 or 2 colors per student
- Paper and pencils
- Coins, 6 nickels and 6 pennies

Preparations before beginning Lesson 1:

- Have crayons, paper, and pencils available.
- Run off one copy of Student Sheets 1 and 2 for each student.
- Have 6 nickels and 6 pennies available for demonstration.

Lesson 1: Number and Number Relations

Say:

Today we are going to talk about fractions. Fractions make it possible to represent numbers between whole numbers. Fractions show different types of relationships. In this lesson, we will look only at part/whole relationships. For example, if I cut a pizza into 4 pieces and give you one piece, you get one-fourth of the pizza. I can show this answer as ¼. The whole pizza contains 4 pieces, and you get one part. When I cut a whole into parts, the parts must be the same size in order to show fractional relationships.

⇨ Give Student Sheet 1.

Say:

Look at the 1st rectangle. What do you notice about it? (It is not cut into parts. It is a whole rectangle. It is one whole, etc.) **Let's look at the 2nd rectangle. What do you notice about it?** (It is cut into 4 pieces. All the pieces are the same size. Each piece is called one-fourth, etc.) **Shade or color in one piece. How much have you shaded?** (one-fourth) **Write the symbol for the amount you have shaded.** (1/4) **Tell me what the 1/4 represents.** (1 part shaded out of 4 parts.) **In a fraction, the top number is called the *numerator*. What is the name for the bottom number?** (denominator) **What is the *denominator* in the fraction 1/4?** (4) **Is 1/4 greater or less than 1 whole rectangle?** (less) **Is it about one whole?** (No. It is a lot smaller.)

Say

Let's look at the 3rd rectangle. What do you notice about it? (It is the same as rectangle 2.) **This time shade in 2 pieces. What fraction did you shade?** (two-fourths, 2/4) Some students may realize that this fraction is also the same as 1/2. If they say 1/2, agree and say 2/4 and 1/2 name the same amount. You will look at 1/2 in rectangle 5. **Which is larger, 1/4 or 2/4 of the rectangle?** (2/4) **How do you know?** (Both are cut into fourths; but in 1/4, we shaded only 1 piece. In 2/4, we shaded 2 pieces, so we have more pieces.) **Which would be larger 1/4 or 3/4 of the rectangle?** (3/4) **Why?** (The pieces are the same size, but I have more pieces.) **Which would be larger, 2/4 or 4/4 of the rectangle?** (4/4) **Why?** (The pieces are the same size, but I have more pieces.)

Say:

Let's look at the 4th rectangle. What do you notice about it? (It is the same size as the others. It has been cut into 3 pieces. All the pieces are the same size and shape.) **Shade in one piece. What fraction did you shade?** (one-third, 1/3) **Which is larger, 1/4 or 1/3 of the rectangle?** (1/3) **How do you know?** (Both rectangles are the same size but rectangle 4 was cut into more pieces, so the pieces have to be smaller.) **Which would be larger, 1/3 or 2/3 of the rectangles?** (2/3) **Why?** (The pieces are the same size, but I have more pieces.) **Which would be larger, 3/3 or one whole rectangle?** (Neither. They are the same.)

Say:

Let's look at the 5th rectangle. What do you notice about it? (It is the same size as the others. It has been cut into 2 pieces. All the pieces are the same size and shape.) **Shade in one (1) piece. What fraction did you shade?** (one-half, 1/2) **Which is larger, 1/2 or 1/4 of the rectangle?** (1/2) **How do you know?** (Both rectangles are the same size, but rectangle 3 was cut into more pieces, so the pieces have to be smaller.) **Which is larger, 1/2 or 2/4 of the rectangle?** (Neither. They are the same size.) **When two fractions name the same amount, they are called *equivalent fractions.* 1/2 and 2/4 are equivalent. Have we talked about any other equivalent fractions?** (Yes, 3/3 and 1.) **Which is larger, 2/3 or 1/2 of the rectangle?** (2/3) **How did you decide this answer?** (This one was harder. The fractions did not have the same numerator or denominator, so I had to look at the pieces to decide.)

Say:

Let's look at the 6th rectangle. What do you notice about it? (It is the same size as the others. It has been cut into 6 pieces. All the pieces are the same size and shape.) **Shade in 2 pieces. What fraction did you shade?** (two-sixths, 2/6) **Which is larger 2/6 or 1/3 of the rectangle?** (Neither. They are the same size.) **What are these 2 fractions called?** (Equivalent fractions.) **Which is larger 1/6 or 1/4 of the rectangle?** (1/4) **How do you know?** (Both rectangles are the same size, but rectangle 6 was cut into more pieces, so the pieces have to be smaller.) **Is 2/6 almost a whole?** (No. It is a lot smaller than one whole.) **How could you use the sixths bar to compare 1/2 and 2/3?** (If I think of 1/2 as 3/6 and 2/3 as 4/6, then I can compare the two fractions.) **Which fraction is greater, 1/2 or 2/3?** (2/3)

Say:

Let's look at the 7th rectangle. What do you notice about it? (It is the same size as the others. It has been cut into 4 pieces. All the pieces are **not** the same size and shape.) **Since the rectangle is not divided into equal parts, we are not going to talk about fractions with it. Remember, when we talked about part-to-whole relationships, we said that the whole must be cut into same size parts.**

Say:

Let's look at the rectangles in the 8th picture. What do you notice about this picture? (There are 2 rectangles. They are the same size as the others. They have been cut into 4 pieces. All the pieces are the same size and shape.) **Shade in 5 pieces. What fraction did you shade?** (five-fourths, 5/4) **What did you have to do to shade in 5 pieces?** (Shade one whole rectangle and one part of the next.) **Which is greater, 5/4 or one (1) whole rectangle?** (5/4) **A fraction that has a numerator that is larger than its denominator is always greater than one (1) and is called an *improper* fraction. Is there another way you could name the fractional amount?**

(I could say one (1) whole and one-fourth or $1\frac{1}{4}$.)

This type of number is called a *mixed number*. It has a whole number part and a fractional part. A fraction that has a numerator and denominator that are the same size is equal to one (1). We will look at improper and proper fractions again in a few minutes.

⇨ In this part, you will work with groups of objects rather than rectangles or other figures. Place one (1) nickel and 5 pennies in front of you. If you have coins available, use them; if not, draw 6 circles on your paper. Write 5¢ on one (1) of them and 1¢ on the others.

Say:

How many coins do I have? (6) **How many nickels do I have?** (1) **Write a fraction that could describe the number of nickels in the set.** (one sixth, 1/6) Remind students that you are talking about the number of coins, not the value of the coins. **How many are pennies?** (5) **Write a fraction that could describe the number of pennies in the set.** (five sixths, 5/6)

⇨ Replace two of the pennies with two nickels.

Say:

How many coins do I have? (6) **How many nickels do I have?** (3) **Write a fraction that could describe the number of nickels in the set. (**three sixths, 3/6) **How many are pennies?** (3) **Write a fraction that could describe the number of pennies in the set.** (three sixths, 3/6)

⇨ Arrange the coins so that the 3 pennies are in one row and the 3 nickels are in another row.

Say:

Could anyone describe another way to name the fractional part of nickels in the set? (Hopefully, some will see that another name for this amount is 1/2.) **The fractions 1/2 and 3/6 are equivalent fractions.**

⇨ Give Student Sheet 2. This activity will give practice on fractional parts. Allow the students to do problems 1 – 4 alone. Come back and talk about each problem.

Answers: 1) B 2) D 3) $\frac{2}{8}$ or $\frac{1}{4}$ 4) $1\frac{1}{8}$ or $\frac{9}{8}$

Say:

Before doing the rest of the page, let's look at ways to compare fractions. We have already learned some ways from working with the rectangles. Can anyone summarize what we found or give an example? (Sample response 1. If the denominators are the same, we just looked at the numerators to see whether we had more pieces. In comparing 3/4 and 1/4, the denominators are the same, but we have more pieces in 3/4, so 3/4 is larger. Sample response 2. When the numerators were the same, we looked at the number of pieces each fraction was divided into. In comparing 1/4 and 1/3, the numerators are the same but in 1/4, the whole has been broken into more pieces, so the pieces are smaller.) **Remember when we had to compare 2/3 and 1/2. The numerators are different and so are the denominators. We could find a common denominator and then we could compare the two fractions.**

⇨ Write $\frac{2}{3}$ and $\frac{1}{2}$ on the board. Draw the pictures.

Say:

Can you think of a denominator that we could use to rewrite both of these fractions? (sixths) **If I think of changing 2/3 to sixths, how many sixths would I have?** (4) **So 4/6 is just another way of writing 2/3. How could I write 1/2 using sixths?** (3/6) **Which is larger, 4/6 or 3/6?** (4/6) **Which fraction was equivalent to 4/6?** (2/3) **So 2/3 is larger than 1/2. Let's now look at problems 5 – 8 on Sheet 2.** Allow students to discuss how they would find each answer. Answers:

5) The size of the pieces is the same, so I would look at the numerator to find the number of pieces. There are more pieces in 2/8.

6) The numerator and denominator are different in the 2 fractions, so I need to think of how I could rewrite both fractions. I could use 15ths for both. 1/3 is the same as 5/15 and 2/5 is the same as 6/15, so 2/5 is larger.

7) The numerators and denominators are different. I could think of 12ths. 2/4 is the same as 6/12 and 2/3 is the same as 8/12. 2/3 is the larger fraction.

8) 1/2 and 2/4 are equivalent fractions. They are the same size.

Say:

In this lesson, we also looked at improper fractions and mixed numbers. *Improper fractions* are fractions that are greater than one (1). A fraction whose numerator and denominator are the same is equal to one (1). Which is larger in an improper fraction, the numerator or the denominator? (numerator) **A *mixed number* is a fraction that is also greater than 1. It has a whole number part and a fractional part. How are 3/2 and 1 and 1/2 alike?** See below.

⇨ Write $\frac{3}{2}$ and $1\frac{1}{2}$ on the board.

Say:

$\frac{3}{2}$ names the same amount as $\frac{2}{2} + \frac{1}{2}$. $1\frac{1}{2}$ is the same as one (1) whole and $\frac{1}{2}$.

$\frac{2}{2}$ and one (1) whole name the same amount. How could I write a mixed number for $\frac{9}{4}$? (I could think of $\frac{9}{4}$ as $\frac{8}{4} + \frac{1}{4}$ or $2\frac{1}{4}$.)

How could I write $\frac{11}{3}$ as a mixed number? (I could think of $\frac{11}{3}$ as $\frac{9}{3} + \frac{2}{3}$ or $3\frac{2}{3}$.) **How could I write $1\frac{1}{5}$ as an improper fraction?** (I could think of 1 as $\frac{5}{5}$ so I have $\frac{5}{5} + \frac{1}{5}$ or $\frac{6}{5}$.)

Have students work problems 9 – 12 on Student Sheet 2.

Answers: 9) 1 and 1/4 10) 4 and 1/6 11) 3/2 12) 14/4

Say:

Equivalent fractions name the same amount. If I give you 4 coins and 2 are nickels, which would I use to name the fractional part that are nickels, 1/2 or 2/4? Often, it doesn't matter, but sometimes we want to give the answer in the simplest form. A fraction is in the simplest form when its numerator and denominator have no common factor other than one (1). So what does this statement mean? Factors are numbers that are multiplied together to get a product. What are the factors of 15? (1, 3, 5, and 15.) **What are the factors of 36?** (1, 2, 3, 4, 6, 9, 18, and 36.) **Do 15 and 36 have any factors in common?** (Yes. 1 and 3.)

⇨ Write 2/4 on the board.

Say:

Do 2 and 4 have any factors in common? (Yes. They both have a 2.) **Is this fraction in the simplest form?** (No.) **To write it in the simplest form, we can divide both the numerator and denominator by the common factor of 2. Our answer is 1/2. Is 1/2 in its simplest form?** (Yes. One (1) and 2 have no common factors other than 1.)

⇨ Have the students work 13 – 16 on Student Sheet 2. Do not worry if the students don't see the greatest common factor. They can divide the numerator and denominator more than one time.

Answers:

13) 4/12 = 1/3. Some students may see automatically that the 4 and 12 have the number *4* in common. Some may see only 2. They would get 2/6. There is still a common factor of 2. Just divide the numerator and denominator again. You still get 1/3.

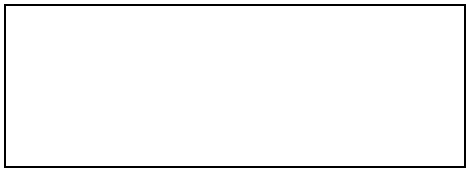
14) 1/2

15) 1/3. Some may see 6, some may see 2, and some may see 3 as the common factor. If they use 2 or 3, they have to divide the numerator and denominator again.
16) 2/3

⇨ Have one student summarize the lesson. In the summary, you want to see whether the students understand the underlying concepts of fractions: equivalency, comparing fractions, mixed numbers and improper fractions, and simplest form.

Student Sheet 1 (Number: Lesson 1)

1.



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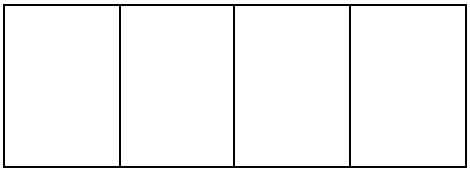

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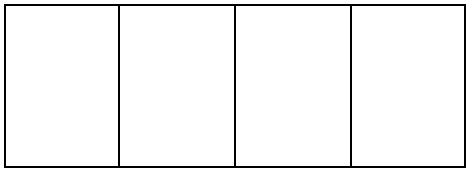

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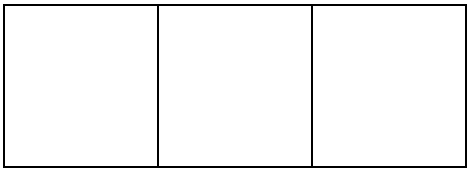

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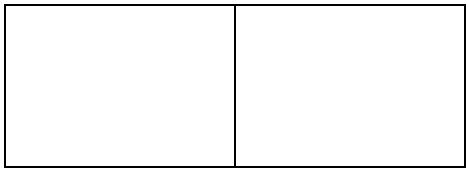

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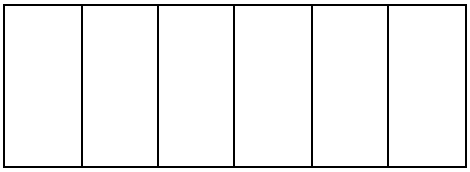

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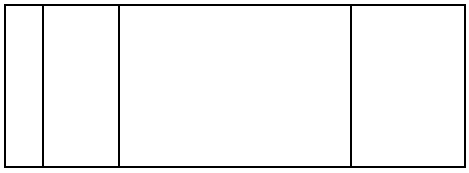

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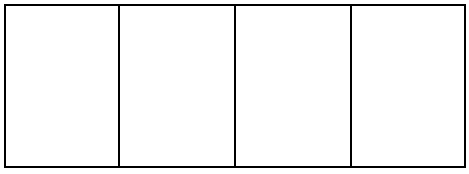
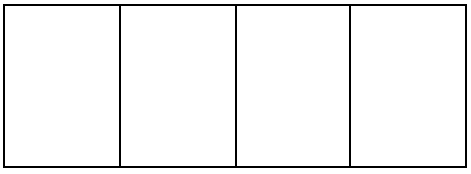
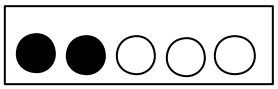
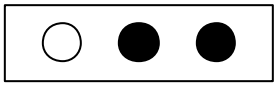




Student Sheet 2 (Number: Lesson 1)

1. Find the picture in which $\frac{2}{3}$ of the circles have been shaded.

A. 

B. 

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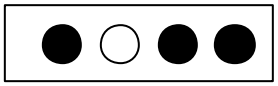
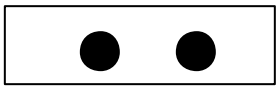
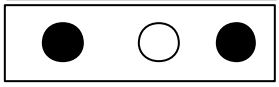
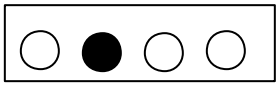
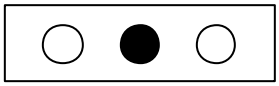
D. 

2. Find the picture in which $\frac{1}{3}$ of the circles have been shaded.

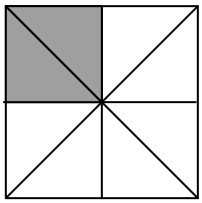
A. 

B. 

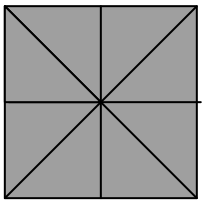
C. 

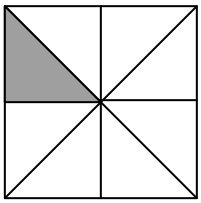
D. 

3. What part of the figure is shaded?



4. How much is shaded? Write as a mixed number and a fraction.





Compare each pair of fractions to decide which is larger. Be ready to explain how you decided.

5. $\frac{1}{8}$ and $\frac{2}{8}$ **6.** $\frac{1}{3}$ and $\frac{2}{5}$ **7.** $\frac{2}{4}$ and $\frac{2}{3}$ **8.** $\frac{1}{2}$ and $\frac{2}{4}$

Write the following improper fractions as mixed numbers.

9. $\frac{5}{4}$ **10.** $\frac{25}{6}$

Write the following mixed numbers as improper fractions.

11. $1\frac{1}{2}$ **12.** $3\frac{2}{4}$

Write the following fractions in simplest form.

13. $\frac{4}{12}$ **14.** $\frac{5}{10}$ **15.** $\frac{6}{18}$ **16.** $\frac{4}{6}$