Teaching For Results

Teacher-to-Teacher
Video Series
for Secondary Educators

TITLE: Getting Familiar with Fractals

PRIMARY SUBJECT AREAS: Mathematics

GRADE LEVELS: 9-10

OVERVIEW: Students will explore the world of fractals by using the Internet to answer lab questions. After discussing the lab results, students will learn how to construct fractals given the initial stage and the iteration rule. Students will then use their knowledge of sequences to complete tables and generate rules for the *nth* term. To wrap up the lesson, students will create their own fractals and put together a fractal book.

APPROXIMATE DURATION: 5 fifty minute class periods

LOUISIANA CONTENT STANDARDS:

http://www/DOE/assessment/standards/MATH.pdf

Algebra

A-1-H demonstrating the ability to translate real-world situations (e.g., distance versus time relationships, population growth, growth functions for diseases, growth of minimum wage, auto insurance tables) into algebraic expressions, equations, and inequalities and vice versa

Geometry

G-1-H identifying, describing, comparing, constructing, and classifying geometric figures in two and three dimensions using technology where appropriate to explore and make conjectures about geometric concepts and figures

G-5-H classifying figures in terms of congruence and similarity and applying these relationships

Measurement

M-2-H demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of results as related to area, volume, mass, rate, and distance)

M-3-H estimating, computing, and applying physical measurement using suitable units (e.g., calculate perimeter and area of plane figures, surface area and volume of solids presented in real-world situations)

M-4-H demonstrating the concept of measurement as it applies to real-world experiences

Discrete Math

D-9-H using discrete math to model real-life situations (e.g., fair games or elections, map coloring)

GLEs Addressed

Grade 9

9. Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
21. Determine appropriate units and scales to use when solving measurement problems (M-2-H) (M-3-H) (M-1-H)

Grade 10

10. Form and test conjectures concerning geometric relationships including lines, angles, and polygons (i.e., triangles, quadrilaterals, and *n*-gons), with and without technology (G-1-H) (G-4-H) (G-6-H)
18. Determine angle measures and side lengths of right and similar triangles using trigonometric ratios and properties of similarity, including congruence (G-5-H) (M-4-H)

EDUCATIONAL TECHNOLOGY GUIDELINES:

http://www/DOE/LCET/curric/k12stand.pdf

Technology Problem Solving and Decision-Making Tools

- Use appropriate technology to locate, retrieve, organize, analyze, evaluate, and communicate information for problem solving and decision-making.

Technology Research Tools

- Demonstrate knowledge and skills of Internet use and other resources consistent with acceptable use policies including the legal consequences of plagiarism and the need for authenticity in student work through and understanding of copyright issues.

Social, Ethical, and Human Issues

- Demonstrate and advocate legal and ethical behaviors among peers, family, and community regarding the use of technology and information

Technology Productivity Tools

- Refine knowledge and enhance skills in keyboarding, word processing, desktop publishing, spreadsheets, databases, multimedia, and telecommunications in preparing and presenting classroom projects.

INTERDISCIPLINARY CONNECTIONS:

Science
Art

OBJECTIVES:

1. The student will explain why fractals are an important field of mathematics.
2. The student will construct fractals given the initial stage and the iteration rule.
3. The student will state the initial stage and the iteration rule given the pictures of at least three stages of a fractal.
4. The student will use computer programs on the Internet to generate numerous stages of fractals that are difficult, if not impossible, to draw by hand.
5. The student will complete tables involving geometric sequences.
6. The student will generate an explicit and a recursive rule for the *nth* term of each sequence.
7. The student will construct his/her own fractal.
8. The student will create a fractal book.

LESSON MATERIALS AND RESOURCES:

Internet Fractal Lab worksheets
Unlined sheets of paper – about 6 sheets per student (colored paper would be nice)
Rulers
Markers or colored pencils

TECHNOLOGY TOOLS AND MATERIALS:

Computers
Web sites

http://www.figleaf.com/development/flash5/koch.swf (Koch Snowflake Generator)
http://www.dennisobrien.net/flash/sierpinski/sierpinski.swf (Sierpinski's Triangle generator)
http://www.arcytech.org/java/fractals/lsystems.shtml (Various fractals)
http://math.rice.edu/~lanius/frac (Fractal notes - teacher)

Calculators - optional

BACKGROUND INFORMATION:

Students should be able to conduct their own Internet searches. In addition, it is helpful if students have been exposed to geometric sequences, explicit formulas, recursive formulas, and special right triangles.

LESSON PROCEDURES:

Day One

1. The students answer questions from the Internet lab in groups of three.

Day Two

1. The students will discuss the lab results.
2. The teacher will begin the construction of fractals with the Cantor Set. Given the initial stage and the iteration rule, the teacher will generate stages 0-2.
3. With limited guidance, the students will work together to generate stage 3 and complete the tables on the Cantor Set.
4. Let the students discuss the results among themselves. Be careful not to give the answers. Let the students come to a consensus on their own.

5. The teacher will begin the construction of the Checkered Flag fractal by stating the initial stage and the iteration rule.
6. With limited guidance, students will work together to generate stages 0-3 and complete the tables on the Checkered Flag fractal. The students may have to finish completing the tables for homework if time runs out.

Day Three

1. Give the students time to discuss the homework before moving on to the next fractal.
2. The teacher will begin the construction of Sierpinski's Triangle, unless a student feels comfortable enough to draw the first couple of stages for the class. This time, the students will try to state the initial stage and the iteration rule.
3. With limited guidance, students will work together to generate stage 3 and complete the tables on Sierpinski's Triangle.
4. Again, provide ample time for discussion among and between the groups before moving on.
5. If you have computers in your classroom, let groups of students generate more stages of Sierpinski's Triangle by visiting the web site: http://www.dennisobrien.net/flash/sierpinski/sierpinski.swf. If you only have one computer, visit the web site and project the images for the students to see.
6. The teacher will begin the construction of the Koch Snowflake, unless a student feels comfortable enough to draw stages 0 and 1 for the class. Again, the students will try to state the initial stage and the iteration rule.
7. With limited guidance, students will work together to generate stage 2 and complete the tables on the Koch snowflake.
8. Allow time for discussion before moving on.
9. Again, allow the students to generate more stages of the Koch Snowflake by visiting the web site: http://www.figleaf.com/development/flash5/koch.swf. If you only have one computer, visit the web site and project the images for the students to see.

Days Four and Five

1. Students will construct their own fractals. Each student must draw at least stages 0-3 and must state the initial stage as well as the iteration rule.
2. Each student will turn in a Fractal Book containing the fractals worked on in class and at least one original creation.

ASSESSMENT PROCEDURES:

Informal Assessment

1. The teacher watches the faces of the students for the "deer in the headlights" look.
2. The teacher walks around to each group to check on progress and ask guiding questions.

Formal Assessment

1. The teacher will grade the Internet lab.
2. The teacher will grade each student's Fractal Book.

ACCOMMODATIONS/MODIFICATIONS:

1. Enlarged print worksheets are needed for visually impaired students.
2. Access to large screen computers are needed for visually impaired students.
3. Students with learning problems may require more time to complete the fractal book.
4. Students with learning problems may require assistance with creating their own fractals.

REPRODUCIBLE MATERIALS:

Internet Fractal Lab (student)
Cantor Set – stages and tables (teacher)
Checkered Flag – stages and tables (teacher)
Sierpinski's Triangle – stages and tables (teacher)
Koch Snowflake – stages and tables (teacher)
Sample Rubric for Fractal Book (teacher)

EXPLORATIONS AND EXTENSIONS:

1. The Square Snowflake
2. The Koch Antisnowflake
3. Geome Tree
4. The Connection Between Pascal's Triangle and Sierpinski's Triangle
5. The Jurassic Park Fractal
6. The Chaos Game
7. The Mandlebrot Set
8. Fractals in art
9. Fractals in nature
10. Fractals in music

LESSON DEVELOPMENT RESOURCES:

Arnold, R., & Tannenbaum, P. (1995) Excursions into Modern Mathematics. Englewood Cliffs: Prentice Hall.

Figleaf. (2002). Koch Snowflake Development [On-line]. Available: http://www.figleaf. Com/development/flash5/koch.swf.

Kamischke, E., Kamischke, E., & Murdock, J. (1998). Advanced Algebra through Data Exploration. Berkeley: Key Curriculum Press.

Lanius. C. (2002). Fractals [On-line]. Available: http://math.rice.edu/~lanius/frac/.

Obrien, D. (2002). Koch snowflake [On-line]. Available: http://www.dennisobrien.net /flash/sierpinski/sierpinski.swf.

Pappas, T. (1993). Fractals, Googols and Other Mathematical Tales. San Carlos: Wide World Publishing/Tetra.

REFLECTIONS:

The students loved this unit! In their groups, students were able to discuss how to complete the tables. Rarely did I have to step in and ask guiding questions. If we had not already completed a unit on sequences and series, the students may have gotten too frustrated to enjoy studying fractals. Some students will need help getting started with creating their own fractals. It really helps to let students share their ideas with each other. One idea can spark many other creations! If you have students that really get into creating their own fractals, they can post their own fractals on the Internet for students all over the country to see.

CONTACT INFORMATION:

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Fractal Internet Lab Sheet

Use the Internet to research fractals. Be sure to include the address of each web site used.

1) What is a fractal?

2) Why are people studying fractals?

3) Name two mathematicians who have studied or who are studying fractals.

4) Approximately how long have people been studying fractals?

5) Name and draw a famous fractal. Be sure to include the initial stage and the rule for generating the fractal.

Fractal Name:

Initial Stage:

Rule:

Stage 0 Stage 1 Stage 2

6) State three real-life applications of fractals. Include pictures or computer print outs if necessary.

7) How might the eroding coastline of Louisiana be aided by the study of fractals?

Cantor Set – Teacher Notes

Initial Stage: Start with a line segment.

Iteration Rule: Remove the middle third of each segment.

Stage 0

Stage 1

Stage 2

Stage 3

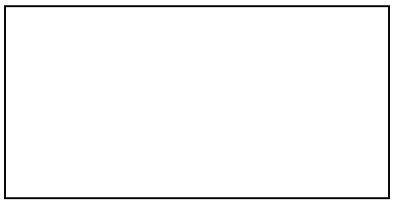
Stage	0	1	2	3	…	Explicit Formula for the nth Term	Recursive Formula for the nth Term
Number of line segments	1	2	4	8	…	$g_n = 2^n$	$g_0 = 1$; $g_n = 2g_{n-1}$
Length of each segment	1	1/3	1/9	1/27	…	$g_n = (1/3)^n$	$g_0 = 1$; $g_n = 1/3\ g_{n-1}$
Total length of all segments	1	2/3	4/9	8/27	…	$g_n = (2/3)^n$	$g_o = 1$; $g_n = 2/3\ g_{n-1}$

Checkered Flag – Teacher Notes

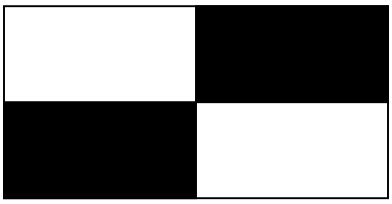
Initial Stage: Start with a white rectangle.

Iteration Rule: Divide each white rectangle into four smaller and congruent rectangles. Darken in the top right and the bottom left rectangles.

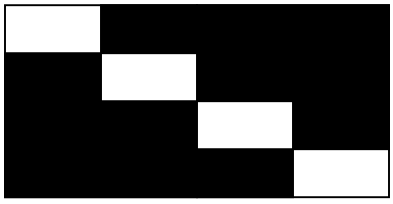
Stage 0



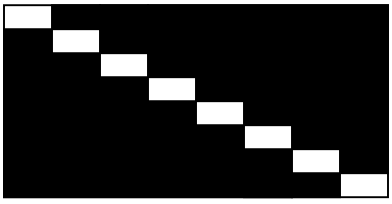
Stage 1



Stage 2



Stage 3



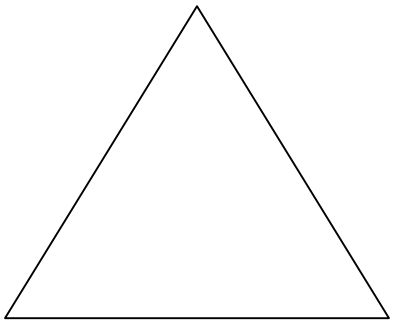
Stage	0	1	2	3	…	Explicit Formula for the nth Term	Recursive Formula for the nth Term
Area of each white rectangle	2	½	1/8	1/32	…	$g_n = 2(1/4)^n$	$g_0 = 2$; $g_n = 1/4\ g_{n-1}$
Perimeter of each white rectangle	6	3	3/2	3/4	…	$g_n = 6(1/2)^n$	$g_0 = 6$; $g_n = 1/2\ g_{n-1}$

Sierpinski's Triangle – Teacher Notes

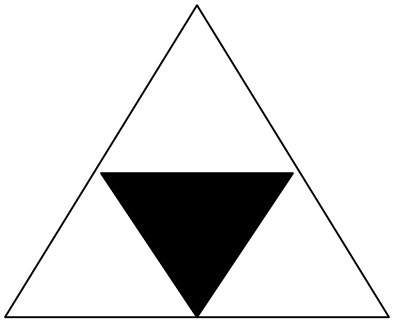
Initial Stage: Start with an equilateral triangle.

Iteration Rule: Connect the midpoints of each side to form four congruent equilateral triangles. Remove the middle triangle by darkening it in.

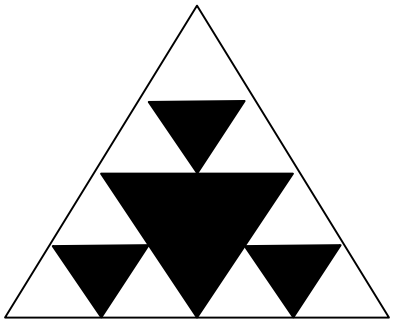
Stage 0



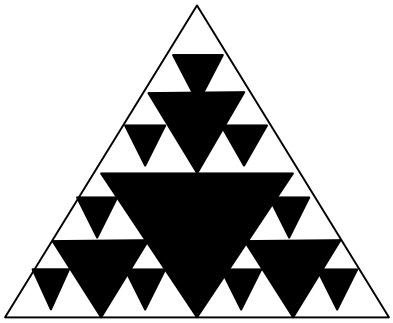
Stage 1



Stage 2



Stage 3



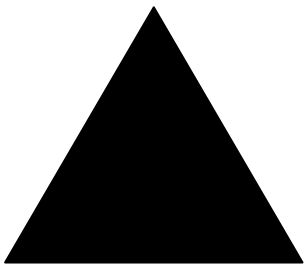
Stage	0	1	2	3	…	Explicit Formula for the nth Term	Recursive Formula for the nth Term
Number of white triangles	1	3	9	27	…	$g_n = 3^n$	$g_0 = 1$; $g_n = 3\, g_{n-1}$
Perimeter of each white triangle	3	3/2	3/4	3/8	…	$g_n = 3(1/2)^n$	$g_0 = 3$; $g_n = 1/2\, g_{n-1}$
Area of each white triangle	$\frac{\sqrt{3}}{4}$	$\frac{\sqrt{3}}{16}$	$\frac{\sqrt{3}}{64}$	$\frac{\sqrt{3}}{256}$	…	$g_n = \frac{\sqrt{3}}{4}\left(\frac{1}{4}\right)^n$	$G_0 = \frac{\sqrt{3}}{4}$, $g_n = \frac{1}{4} g_{n-1}$

Koch Snowflake – Teacher Notes

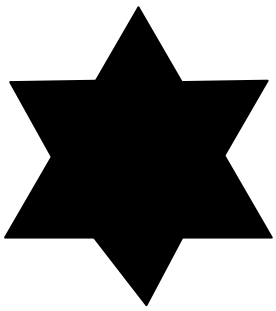
Initial Stage: Start with an equilateral triangle.

Iteration Rule: Divide each side into three equal segments. Attach an equilateral triangle to the middle segment of each side.

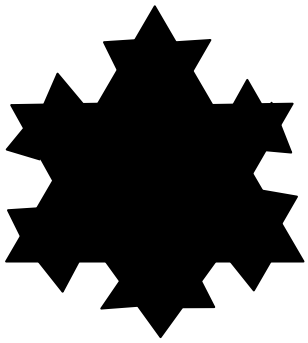
Stage 0



Stage 1



Stage 2



Stage	0	1	2	…	Explicit Formula for the nth Term	Recursive Formula for the nth Term
Length of each side	1	$\frac{1}{3}$	$\frac{1}{9}$	…	$g_n = \left(\frac{1}{3}\right)^n$	$g_0 = 1$; $g_n = \frac{1}{3} g_{n-1}$
Number of sides	3	12	48	…	$g_n = 3(4)^n$	$g_0 = 3$; $g_n = 4\, g_{n-1}$
Perimeter of the snowflake	3	4	$\frac{16}{3}$	…	$g_n = 3\left(\frac{4}{3}\right)^n$	$g_0 = 3$; $g_n = \frac{4}{3} g_{n-1}$

Sample Rubric for the Fractal Book

Introduction

- Definition of Fractals (5 pts)
- Brief background (4 pts)
- Current applications (4 pts)

Cantor Set

- Initial Stage Description (1 pt)
- Iteration Rule (2 pts)
- Drawing of stages 0-3 (4 pts)
- Completed tables (6 pts)

Checkered Flag

- Initial Stage Description (1 pt)
- Iteration Rule (2 pts)
- Drawing of stages 0-3 (4 pts)
- Completed tables (4 pts)

Sierpinski's Triangle

- Initial Stage Description (1 pt)
- Iteration Rule (2 pts)
- Drawing of stages 0-3 (4 pts)
- Completed tables (6 pts)

Koch Snowflake

- Initial Stage Description (1 pt)
- Iteration Rule (2 pts)
- Drawing of stages 0-3 (4 pts)
- Completed tables (6 pts)

Original Fractal

- Initial Stage Description (1 pt)
- Iteration Rule (2 pts)
- Drawing of stages 0-3 (4 pts)

Total Points Possible = 70