Teacher-to-Teacher
Video Series
for Secondary Educators

TITLE: Translating Transformations in Geometry

PRIMARY SUBJECT AREA: Geometry

OVERVIEW: In this lesson we will examine three basic transformations in the coordinate plane. Transformations (also known as mappings) are geometric figures that have been changed by reflection, rotation, or translation (also known as flips, turns, and slides). Numerical connections between the figures and the coordinate plane will be established and patterns will be recognized between the preimage and the image.

APPROXIMATE DURATION: 2 –3 fifty minute class periods

LOUISIANA CONTENT STANDARDS:

http://www/DOE/assessment/standards/MATH.pdf

Geometry

G-1-H identifying, describing, comparing, constructing, and classifying geometric figures in two and three dimensions using technology where appropriate to explore and make conjectures about geometric concepts and figures

G-3-H solving problems using coordinate methods, as well as synthetic and transformational methods (e.g., transform on a coordinate plane a design found in real-life situations)

G-4-H using inductive reasoning to predict, discover, and apply geometric properties and relationships (e.g., patty paper constructions, sum of the angles in a polygon)

GLEs Addressed

Grade 9

26. Perform translations and line reflections on the coordinate plane (G-3-H)

Grade 10

10. Form and test conjectures concerning geometric relationships including lines, angles, and polygons (i.e., triangles, quadrilaterals, and *n*-gons), with and without technology (G-1-H) (G-4-H) (G-6-H)
14. Develop and apply coordinate rules for translations and reflections of geometric figures (G-3-H)

EDUCATIONAL TECHNOLOGY GUIDELINES:
http://www/DOE/LCET/curric/k12stand.pdf

INTERDISCIPLINARY CONNECTIONS:
Art
Language Arts
Family and Consumer Science
Agriscience

OBJECTIVES:
1. The student will recognize transformations by each of two terms.
2. The student will visualize the movement of images on the coordinate plane.
3. The student will identify points of the image on the coordinate plane.
4. The student will organize data in the transformation of various images.
5. The student will establish numerical patterns in the transformation of various images.

LESSON MATERIALS AND RESOURCES:
Grid paper
Data Collection Worksheet provided

TECHNOLOGY TOOLS AND MATERIALS:
Geometer's Sketchpad
TI-92 Calculator

BACKGROUND INFORMATION:
Students should be able to identify points on the coordinate plane. When using technology, it is mandatory that the student be able to interpret the technological findings to the data being collected to establish the patterns of the transformations. Data collection methods and analysis are necessary.

LESSON PROCEDURES:
1. The first step in teaching transformations is to clarify terminology. Just as in languages, many words can mean the same thing. Examples could include cold - chilled; tasteful - delicious; beautiful - lovely. The geometric term transformation is sometimes referred to as mapping. Other corresponding terms in this topic are reflection to flip, rotation to turn, and translation to slide. Flip, turn, and slide are the motions made as each transformation is complete. Another pair of terms unique to mapping is preimage (pre-image) and image. The original figure (before transformation) is the preimage and the corresponding figure after the transformation is the image.
2. Have students create the preimage QUAD using the following coordinates:
   Q (4,8)
   U (8,7)
   A (6,1)
   D (2,4)

Tactile learners should cut out the preimage and manipulate it on the coordinate plane. Other students may need only to observe the demonstration.

3. Translate (slide) QUAD as indicated in Table 1 on the Transformation Worksheet. Record the new coordinates after each move. Always return to the preimage before making the next move. The solutions are given in Teacher's Table 1.
4. After we have established the coordinates of our 4 moves, we will evaluate the change in the x coordinate from the preimage to the image, then the same for the y coordinate. Lead students to observe that when the slide involves only one direction (either up/down or left/right), the image is moving along a vertical or horizontal line. Moves that involve two directions up or down and left or right, create the image on a line diagonal from the preimage.
5. In our next transformation we will reflect an image. Reflections are a flip over a line on the plane. Teacher's Table 2 provides solutions.
6. The points after a reflection are labeled as Q'U'A'D' (Q prime, U prime, etc.) and after the second reflection as Q"U"A"D" (Q double prime, U double prime, etc.)
7. Students should notice that in a reflection over the y-axis the position of the points in the quadrilateral change from left to right and that over the *x*-axis they change from top to bottom. When the figure is reflected over both axis, the top and bottom switch and the left and right switch.
8. That brings us to our last transformation: the rotation. A rotation is the movement of a figure around a given point on the plane. To simplify this move we will place QUAD in a position where one of its vertices is at the origin. (We are using the same quadrilateral just in a different position.) When physically moving the figure, move the leading edge the number of degrees indicated. In our diagram the leading edge is on the *y*-axis so each 90D rotation will stop on the next axis. See Teacher's Table 3 for the solution.
9. The pattern I see developing in the rotation is that the *x*- and *y*-coordinates change places with each 90º rotation. After several experiments I have discovered that when moving clockwise the sign of the *y*-coordinate will change with each 90º rotation and counter-clockwise moves will change the sign of the *x*-coordinate with each 90º rotation.
10. As I tried to remember which of these rules is which, I made up this anecdote:
    Wise (y's) people move in the right direction (clockwise) while people going in the wrong direction (counterclockwise) need to examine (x-amine) where they're headed
11. I am also noticing that a 180º rotation produces the same results as two flips (reflections) over the axis…in other words, the reflection of a reflection.

12. Since rotation seems to be the most complicated of our transformations, let's work in reverse to see if we can diagram a figure as it is rotated around the origin. Our figure has 3 points:
    S (-2,7)
    U (5,3)
    N (6,8)
13. Using the rules we've established, complete the points on the chart provided on your worksheet. (Teacher's Table 4 is the teacher's key.)
14. Using the chart you've developed in the previous step, plot the points on your coordinate plane and watch the triangle SUN rotate around the origin.
15. Have students complete the Symmetry Project.
16. Hopefully, these basic instructions explaining translations, reflections, and rotations will make you more confident when you slide, flip, and turn in your geometry class.

ASSESSMENT PROCEDURES:

Assessment should include the transformation worksheet (even though this was done together using it as an assessment is a motivation for students to stay engaged). Practice diagrams should be given on the coordinate plane and evaluated for all transformations. Informal assessments should be done by showing transformations of figures and having students determine which transformation has been accomplished. The Symmetry Project is the ultimate assessment of reflection and translations. A rubric for the symmetry project is provided.

ACCOMMODATIONS/MODIFICATIONS:

Tactile learners are encouraged to manipulate figures on the coordinate plane.

REPRODUCIBLE MATERIALS:

Transformation Worksheet
Coordinate Grid
Symmetry Project and rubric

EXPLORATIONS AND EXTENSIONS:

Students could enhance the skills introduced in this lesson by creating tessellations (translation/slide), symmetrical patterns created on a grid (reflection/flip), or a kaleidoscope (rotation/turn).

LESSON DEVELOPMENT RESOURCES:

Patty Paper Geometry by Michael Serra

REFLECTIONS:

These are some of my favorite lessons to teach to my students. I have discovered that MANY students know what "area" is, but have little understanding of area connections and applications. Many of them only know "area" as a formula. The circle activity is to geometry what dissection is to biology! The students are amazed at the discovery and you will usually have one

or two who want to cut the circle into more sectors. I've them cut up to 64 "slivers"! It's exciting to me to see them become excited over mathematics!

CONTACT INFORMATION:

Lisa Taylor
West Monroe High School
taylor.lisa@excite.com

TRANSFORMATION WORKSHEET

TABLE 1 - TRANSLATIONS						
Movement instructions	**Q**	**U**	**A**	**D**	**ΔX**	**ΔY**
PREIMAGE	**(4,8)**	**(8,7)**	**(6,1)**	**(2,4)**	**N/A**	**N/A**
Down 4						
Left 4						
Down 4 and Left 4						
Down 4 and Right 3						

TABLE 2 - REFLECTIONS						
Movement instructions	**Q**	**U**	**A**	**D**	**ΔX**	**ΔY**
PREIMAGE	(4,8)	(8,7)	(6,1)	(2,4)	N/A	N/A
Reflect over y axis						
Reflect over x axis						
Reflect over x axis then y axis						

TABLE 3 - ROTATIONS						
Movement instructions	**Q**	**U**	**A**	**D**	**ΔX**	**ΔY**
PREIMAGE	**(5,6)**	**(6,2)**	**(0,0)**	**(0,5)**	**N/A**	**N/A**
90º counterclockwise						
180º counterclockwise						
270º counterclockwise						
360º counterclockwise						

TABLE 4 - ROTATIONS			
Movement instructions	**S**	**U**	**N**
PREIMAGE	(-2,7)	(5,3)	(6,8)
90º counterclockwise			
180º counterclockwise			
270º counterclockwise			
360º counterclockwise			

TEACHER'S TRANSFORMATION WORKSHEET

TEACHER'S TABLE 1						
Movement instructions	**Q**	**U**	**A**	**D**	**ΔX**	**ΔY**
PREIMAGE	(4,8)	(8,7)	(6,1)	(2,4)	N/A	N/A
Down 4	(4,4)	(8,3)	(6,-3)	(2,0)	None	Subtract 4
Left 4	(0,8)	(4,7)	(2,1)	(-2,4)	Subtract 4	None
Down 4 and Left 4	((0,4)	(4,3)	(2,-3)	(-2,0)	Subtract 4	Subtract 4
Down 4 and Right 3	(7,4)	(11,4)	(9,-3)	(5,0)	Add 3	Subtract 4

TEACHER'S TABLE 2						
Movement instructions	**Q**	**U**	**A**	**D**	**ΔX**	**ΔY**
PREIMAGE	(4,8)	(8,7)	(6,1)	(2,4)	N/A	N/A
Reflect over y axis	(-4,8)	(-8,7)	(-6,1)	(-2,4)	Opposite of preimage x	None
Reflect over x axis	(4,-8)	(8,-7)	(6,-1)	(-2,4)	None	Opposite of preimage y
Reflect over x axis then y axis	(-4, -8)	(-8,-7)	(-6,-1)	(-2,-4)	Opposite of preimage x	Opposite of preimage y

TEACHER'S TABLE 3						
Movement instructions	**Q**	**U**	**A**	**D**	**ΔX**	**ΔY**
PREIMAGE	(5,6)	(6,2)	(0,0)	(0,5)	N/A	N/A
90º counterclockwise	(-6,5)	(-2,6)	(0,0)	(-5,0)	Opposite of preimage y	Equal to preimage x
180º counterclockwise	(-5,-6)	(-6,-2)	(0,0)	(0,-5)	Opposite of preimage x	Opposite of preimage y
270º counterclockwise	(6,-5)	(2,-6)	(0,0)	(5,0)	Equal to preimage y	Opposite of preimage x
360º counterclockwise	(5,6)	(6,2)	(0,0)	(0,5)	Equal to preimage x	Equal to preimage y

TEACHER'S TABLE 4			
Movement instructions	**S**	**U**	**N**
PREIMAGE	(-2,7)	(5,3)	(6,8)
90º counterclockwise	(7,2)	(3,-5)	(8,-6)
180º counterclockwise	(2,-7)	(-5,-3)	(-6,-8)
270º counterclockwise	(-7,-2)	(-3,5)	(-8,6)
360º counterclockwise	(-2,7)	(5,3)	(6,8)

SYMMETRY PROJECT

Use the rough draft worksheet to make up your own pattern for 4 individual squares. No two squares may be alike. You will use ONLY these four squares for creating your project. Follow the pattern shown below to create your design. When turned in, your rough draft must be colored on the worksheet provided.

ROUGH COPY DUE DATE_____

After your rough copy is approved, you will transfer your design to a 10" x 10" poster board. The final product is square with a 1-inch border around the design. This border must be drawn in ink or marker. You must measure and lightly mark off the border and squares before you start.

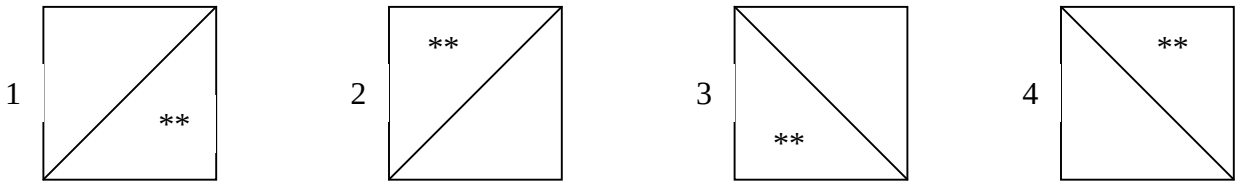
You must color your final project the same way you colored your original design. Be sure to follow the approved pattern. The rough draft must be attached to the back of the final project. You may not change your pattern after the rough draft has been checked.

Any pencil lines used for drawing the pattern should be very light. Erase any lines not needed for the final design. The final project should be as neat as possible.

NEATNESS COUNTS FOR A LARGE PORTION OF YOUR FINAL GRADE.

FINAL PROJECT DUE DATE_____

To make a design, give each number in the table a pattern. This is just an example of four simple patterns.



** Indicates a shaded area in the design

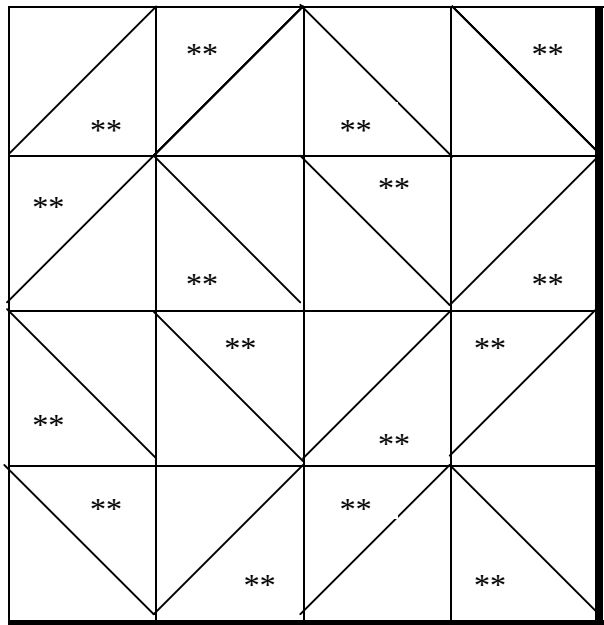
1	2	3	4
2	3	4	1
3	4	1	2
4	1	2	3

To create your original design, place your patterned squares according to the numbers in the table shown here.

Symmetry Project Pattern

The example below is 8 squares by 8 squares

Original design



Original design reflected over vertical line of symmetry

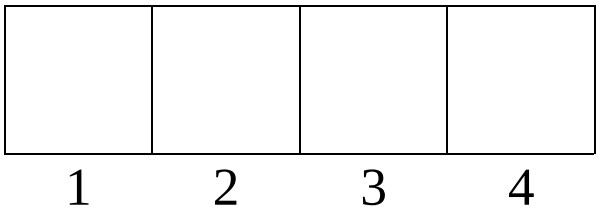
Original design reflected over horizontal line of symmetry

Original design reflected over horizontal and vertical lines of symmetry

** Indicates a shaded area in the design

Name_____ Date_____

Symmetry Project Rough Draft



Symmetry Rubric

Rough Draft on time		10
Rough Draft turned in		10
Correct pattern indicated		10
10x10 grid accuracy		10
Lines erased		10
Border drawn		10
Symmetry accuracy		20
Neatness		20
TOTAL POINTS EARNED		100

Symmetry Rubric

Rough Draft on time		10
Rough Draft turned in		10
Correct pattern indicated		10
10x10 grid accuracy		10
Lines erased		10
Border drawn		10
Symmetry accuracy		20
Neatness		20
TOTAL POINTS EARNED		100

Symmetry Rubric

Rough Draft on time		10
Rough Draft turned in		10
Correct pattern indicated		10
10x10 grid accuracy		10
Lines erased		10
Border drawn		10
Symmetry accuracy		20
Neatness		20
TOTAL POINTS EARNED		100

Symmetry Rubric

Rough Draft on time		10
Rough Draft turned in		10
Correct pattern indicated		10
10x10 grid accuracy		10
Lines erased		10
Border drawn		10
Symmetry accuracy		20
Neatness		20
TOTAL POINTS EARNED		100