TITLE: Connecting Formulas Related to Geometric Figures

PRIMARY SUBJECT AREA: Geometry

OVERVIEW: This lesson will attempt to show students connections among formulas used in geometry. The concentration will be on the area formulas of quadrilaterals and circles.

APPROXIMATE DURATION: 2 –3 fifty minute class periods

LOUISIANA CONTENT STANDARDS:
http://www/DOE/assessment/standards/MATH.pdf

Geometry

G-1-H identifying, describing, comparing, constructing, and classifying geometric figures in two and three dimensions using technology where appropriate to explore and make conjectures about geometric concepts and figures

G-4-H using inductive reasoning to predict, discover, and apply geometric properties and relationships (e.g., patty paper constructions, sum of the angles in a polygon)

Measurement

M-2-H demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of results as related to area, volume, mass, rate, and distance)

Algebra

A-5-M demonstrating the connection of algebra to the other strands and to real-life situations

GLEs Addressed

Grade 9

22. Determine appropriate units and scales to use when solving measurement problems (M-2-H) (M-3-H) (M-1-H)

Grade 10

10. Form and test conjectures concerning geometric relationships including lines, angles, and polygons (i.e., triangles, quadrilaterals, and *n*-gons), with and without technology (G-1-H) (G-4-H) (G-6-H)

17. Compare and contrast inductive and deductive reasoning approaches to justify conjectures and solve problems (G-4-H) (G-6-H)

EDUCATIONAL TECHNOLOGY GUIDELINES:

http://www/DOE/LCET/curric/k12stand.pdf

INTERDISCIPLINARY CONNECTIONS:

Art

OBJECTIVES:

1. The student will relate each area formula to a very basic formula.
2. The student will recognize various ways to express the same formula.
3. The student will realize that most figures are easily manipulated into another.

LESSON MATERIALS AND RESOURCES:

Large printed circle
Scissors
Colored pencils/markers
Scrap paper

TECHNOLOGY TOOLS AND MATERIALS:

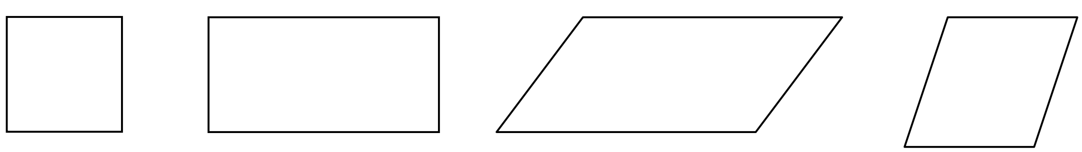
Geometer's Sketchpad
TI-92 Calculator

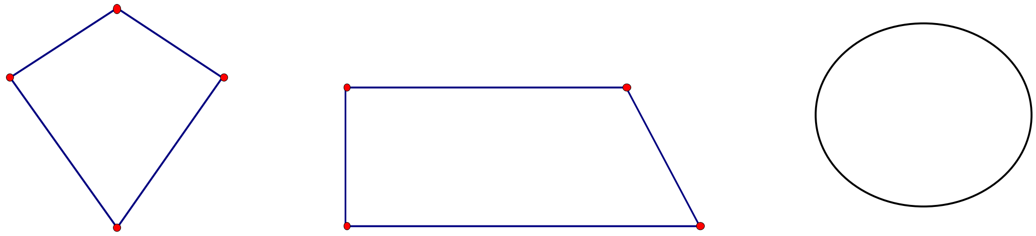
BACKGROUND INFORMATION:

Students should be able to recognize quadrilaterals (parallelograms, rectangles, trapezoids, rhombi, kites, etc.) as well as a circle. Students should be familiar with the properties of quadrilaterals.

LESSON PROCEDURES:

1. Display diagrams of quadrilaterals and circles. Have students name each figure.





2. Have students identify each figure by every possible name (i.e. the square is a quadrilateral, parallelogram, rectangle, rhombus, and square).
3. Have students classify the figures. There are 4 parallelograms and only one kite, trapezoid and circle.
4. As students classify, ask them why certain figures are or are not in that classification. (Students have a hard time realizing that trapezoids and kites are not parallelograms.)
5. Ask students to name the area formula for each diagram. (The students may not know all of these at this time.) They will be confused as they try to tell you some of the formulas. This lesson is an attempt to eliminate that confusion.
6. Begin with the rectangle. Ask students to show you with their hands the width of a television screen. Ask them to show you the length of a foot. In each case, their hands will be spaced horizontally. Then pose the question that if the area of a rectangle is *length* • *width*, how do you determine which is which? If length is longer, then what is width in the #2 diagram shown? I am proposing that we use the formula *base* • *height* where the base is always the bottom or foundation of the figure and the height is always the perpendicular distance from bottom to top. (At this point, tactile students may need to have cutouts of the quadrilaterals being considered. Visual and auditory learners will be able to follow the teacher's instructions.)
7. Show students that if you replaced "inches" with *x,* then the equation would be

$$2x \cdot 2x =$$

$$2 \cdot x \cdot 2 \cdot x =$$

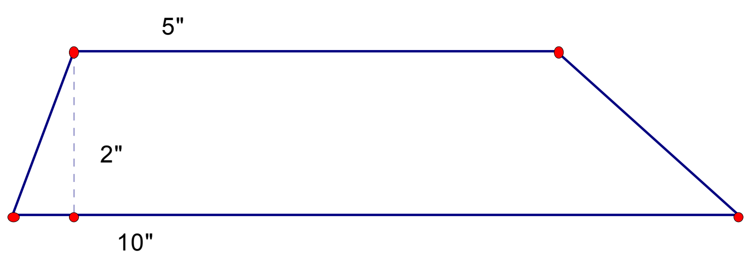
$$2 \cdot 2 \cdot x \cdot x = 4x^2$$

   They are familiar with multiplying variables but see no relationship to *inches* • *inches* = *inches²*.
8. The area in figure two obviously is computed as $base \cdot height$ as the height is perpendicular to the base.
9. Figure 5 is an interesting shape called a kite. Some geometry books do not even address this quadrilateral but it has some interesting qualities:
   - It has 2 distinct pairs of congruent, adjacent sides.
   - The diagonals of the kite are always perpendicular.
   - Exactly one pair of opposite angles of a kite is bisected by a diagonal.
   - Exactly one pair of opposite angles of a kite is congruent.
   - Exactly one diagonal of a kite bisects the other.

10. So, how can we use our old faithful formula to find a kite's area? Cut the kite along the diagonal that has been bisected by the other. This will form two unique isosceles triangles.

11. Manipulate each triangle individually by cutting along the other diagonal of the kite then rotate each right triangle to form a rectangle.

12. Now, by using "old faithful" and a little addition of two area values you can determine the area of a kite. The beauty of this method is that you can use the Pythagorean Theorem to find any unknown sides.
13. Attempt to do the same maneauver in diagram 6 (the trapezoid). (Be sure to use an irregular trapezoid for your example. An isosceles trapezoid will work just like a parallelogram.) Students will see that there is no fit of the triangle to the opposite end. This is a great time to lead them in discovery learning. The trapezoid is a somewhat new quadrilateral to them. Let them explore ways to find the area.
14. One method that students may discover is to cut a triangle off the end of the trapezoid, find the area of the triangle and the area of the remaining rectangle then find their sum. This is certainly a legitimate method if there is enough information.
15. But what if our trapezoid looks like this: Let's call this diagram 8 and use it to discover a formula that will work for every trapezoid!



16. Manipulate the trapezoid every way possible to convince students that it just can't be cut and manipulated to form a parallelogram (like our other examples have). Suggest to the students that "cutting" a quadrilateral is a way of "dividing" it so if the trapezoid can't be "divided" to find a formula what would happen if we "multiplied"?
17. Use colored markers to indicate each side of the trapezoid then replicate the trapezoid on a transparent sheet in the same colors. Rotate the 2nd trapezoid until the two form a parallelogram. Students will now recall the method of cutting a parallelogram on its height and sliding it to form a rectangle.
18. From here we can use "old faithful": $area = b \cdot h$. Let's say that our bases are red and blue and the height is green. Start with the colors:

$$\text{Area} = b \cdot h$$

$$\text{Area} = (blue + red) \cdot green$$

$$\text{Area} = (10" + 5") \cdot 2"$$

$$\text{Area} = (15") \cdot 2"$$

$$\text{Area} = 30\,inches^2$$

19. BUT we must remember what we did to determine the formula…we had to double (replicate) the trapezoid. How do we undo that? Divide by 2 or multiply by ½? AND how do we write in the formula about the top and bottom bases that we call blue and red? We'll call them base_1 and base_2 (b_1 and b_2).
20. SO here goes:

$$\frac{[(b_1 + b_2) \cdot h]}{2} = \text{area of trapezoid}$$

or

$$\tfrac{1}{2}(b_1 + b_2) \cdot h = \text{area of trapezoid}$$

or

$$(b_1 + b_2) \cdot \frac{h}{2} = \text{area of trapezoid}$$

or

$$(b_1 + b_2) \cdot \tfrac{1}{2} h = \text{area of trapezoid}$$

21. WHEW! That took forever but is it worth it to help students understand the foundation of area…I think so!
22. SO how does the circle fit into this scheme? Every student will want to participate in this tactile activity. I find it easier to give every student a pre-drawn circle with a thick circumference drawn as large as possible on a piece of paper. They can certainly use a compass and draw one, but emphasize that it must be as large as possible. Colored pencils and markers are a great asset to this project.
23. Students must know the part of the circle we now have - circumference. Just like "old faithful," $b \cdot h$ = area of parallelogram; the formula for circumference is our basic formula for a circle. We would have previously

experimented to find the ratio for $\pi = \frac{c}{d}$. Using algebra skills, we would solve for $c = \pi d$.

24. Students should fold a diameter of the circle. Use a wide marker or thickly drawn line to mark it. Cut through the middle of the thick line exactly on the diameter.
25. Review terms related to circles through each step of the exercise. Fold each semi-circle in half on the diameter forming a radius of the circle.
26. Fold each arc of the quarter circle half forming another radius of the circle. Cut along each fold. You now have 8 sectors of the circle, with each side of the central angle a radii of the circle.
27. Once more, fold each of the arcs of the 8 sectors in half. Cut along each fold. You now have 16 sectors of the original circle.
28. Students should locate 2 pieces that have one side colored like the original diameter. Set them aside.
29. Assemble the remaining 14 pieces with the first vertex up, the second vertex down, etc. Finally, add the two pieces that were set aside on each end.
30. Students will see that what once was perfectly round is now almost a parallelogram - I call it a "kinda-sorta-parallelogram" (k-s-p). Analyze the perimeter of the k-s-p. The kinda-sorta height is the radius of the circle (the more sectors you cut the closer the slant height would equal the actual height).
31. Ask students to identify the part of the circle that is now the base of the k-s-p. They will always say that it is the circumference. Lead them to understand that it is only half of the circumference.
32. Then using the old faithful formula, *area* $= b \cdot h$, substitute the parts of the circle in the formula"

$$\text{Area} = b \cdot h$$

$$\text{Area} \approx \tfrac{1}{2} c \cdot r$$

NOTE: Remember that this is not a precise parallelogram so our = becomes ≈.

33. Through our previous discovery of π and our ability to solve literal equations, we know that $c = \pi d$ so substitute that in the formula:

$$\text{Area of a parallelogram} = b \cdot h$$

$$\text{Area of a circle} \approx \tfrac{1}{2} c \cdot r$$

$$\text{Area of a circle} \approx \tfrac{1}{2} (\pi d) \cdot r$$

34. And because we know the parts of a circle, we know that $d = 2r$, so substitute that:

$$\text{Area of a circle} \approx \tfrac{1}{2} (\pi d) \cdot r \ .$$

35. Then solve the equation breaking each term into separate pieces and then grouping like terms to multiply:

$$\text{Area of a circle} \approx \tfrac{1}{2} \cdot \pi \cdot 2 \cdot r \cdot r$$

$$\text{Area of a circle} \approx \tfrac{1}{2} \cdot 2 \cdot \pi \cdot r \cdot r$$

$$\text{Area of a circle} \approx 1 \cdot \pi \cdot r^2$$

$$\text{Area of a circle} \approx \pi r^2$$

36. TA-DAH!!! Each shape we have examined today can be related back to one old faithful formula $area = base \cdot height$ so why fret when you're asked to find the area of anything? Every quadrilateral is just a manipulated parallelogram and even a circle is a kinda-sorta-parallelogram with a different view on the world!

ASSESSMENT PROCEDURES:

1. Observation assessment is key in this lesson. All students should be engaged and participating in the discovery of the formulas.

2. Formal assessment can be accomplished through various problems involving area of quadrilaterals and circles.
3. Students should be able to write a paragraph comparing the area of any two quadrilaterals and/or describing the relationship between the area of quadrilaterals and circles.

ACCOMMODATIONS/MODIFICATIONS:

Tactile students should cut and manipulate each figure with teacher. All students will benefit from that method in finding the area of the circle.

REPRODUCIBLE MATERIALS:

One thickly-drawn, large circle.

EXPLORATIONS AND EXTENSIONS:

Prior to beginning the lessons, students should review methods for solving literal equations. I like to have students circle the variable for which they are solving and draw a vertical line through the equal symbol to divide the two sides of the equation.

Prior to beginning the circle lesson, students should measure the diameter and circumference of several circular objects. By setting up their data in a table, they can solve to find the approximate value of π. This understanding is very helpful in the area lesson.

LESSON DEVELOPMENT RESOURCES:

Patty Paper Geometry by Michael Serra

REFLECTIONS:

These are some of my favorite lessons to teach to my students. I have discovered that MANY students know what "area" is, but have little understanding of area connections and applications. Many of them only know "area" as a formula. The circle activity is to geometry what dissection is to biology! The students are amazed at the discovery and you will usually have one or two who want to cut the circle into more sectors. I've them cut up to 64 "slivers"! It's exciting to me to see them become excited over mathematics!

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