Teaching For Results

# Teacher-to-Teacher
Video Series
for Secondary Educators

## TITLE: Slopes of Lines

**PRIMARY SUBJECT AREAS:** Algebra-Linear Equations

**GRADE LEVELS:** 8-12

**OVERVIEW:** This lesson will introduce the concept of the slope of lines as a rate of change by examining the relationship between slope, linear equations, and their graphs. As the lesson begins, students examine a graph and relate it to the real world situation it depicts. Using a graphing calculator in the next activity, students investigate the relationship between changes in the equation and changes in the appearance of the line. The last activity allows the students to investigate the connection between the graph of a situation and the meaning of the slope as it applies to that situation. The real world meaning of the slope is emphasized.

**APPROXIMATE DURATION:** 3 fifty minute class periods

## LOUISIANA CONTENT STANDARDS:
http://www/DOE/assessment/standards/MATH.pdf

**Algebra**

A-1-H demonstrating the ability to translate real-world situations (e.g., distance versus time relationships, population growth, growth functions for diseases, growth of minimum wage, auto insurance tables) into algebraic expressions, equations, and inequalities and vice versa

A-2-H recognizing the relationship between operations involving real numbers and operations involving algebraic expressions

A-3-H Using tables and graphs as tools to interpret algebraic expressions, equations, and inequalities.

**Patterns, Relations, and Functions**

P-1-H modeling the concepts of variables, functions, and relations as they occur in the real world and using the appropriate notation and terminology

P-2-H Translating between tabular, symbolic, or graphic representations of functions

P-3-H recognizing behavior of families of elementary functions, such as polynomial, trigonometric, and exponential functions, and, where appropriate, using graphing technologies to represent them

P-4-H Analyzing the effects of changes in parameters on the graphs of functions, using technology whenever possible.

**GLEs Addressed**

Grade 9

9. Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
13. Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)
15. Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
25. Explain slope as a representation of "rate of change" (A-1-H) (G-3-H)
37. Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)
38. Identify and describe the characteristics of families of linear functions, with and without technology (P-3-H)
39. Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)
40. Explain how the graph of a linear function changes as the coefficients or constants are changed in the function's symbolic representation (P-4-H)

## EDUCATIONAL TECHNOLOGY GUIDELINES:

http://www/DOE/LCET/curric/k12stand.pdf

**Technology Productivity Tools**

- Students use technology tools to enhance learning, increase productivity, and promote creativity.

## INTERDISCIPLINARY CONNECTIONS:

N/A

## OBJECTIVES:

1. The student will identify changes in slope on a graph and relate their meaning to the situation graphed.
2. The student will express the slope of a line as a rate of change.
3. The student will identify horizontal lines as having a zero slope (rate of change) and vertical lines as having no slope (rate of change).
4. The student will be able to identify the slope of a line from its graph and its equation.
5. The student will be able to describe the relationship between the graph, the equation, and the slope of a line.
6. The student will use appropriate units when identifying the slope of a graphed line.

## LESSON MATERIALS AND RESOURCES:

Overhead transparency of Bathtub Problem or individual copies for each student
Teacher Notes for Bathtub Problem Follow-up Discussion
Teacher Notes for Investigating the Relationship Between Linear Equations and Their Graphs
Investigating the Relationship Between Linear Equations and Their Graphs Worksheet
Meaning of Slope: Rate of Change in Graphs Worksheet
Worksheet Answer Key
Algebra textbook

## TECHNOLOGY TOOLS AND MATERIALS:

Overhead graphing calculator
Graphing calculator for each student

## BACKGROUND INFORMATION:

Students should recognize and understand rates, such as miles per hour or cost per person. Students should have basic knowledge of linear equations. They should have experience in graphing linear equations, both with paper and pencil and with a graphing calculator. They should be knowledgeable in interpreting various types of graphs. Students should have had a basic introduction to slope and be able to identify slopes by the formula, slope = $\frac{rise}{run}$.

## LESSON PROCEDURES:

1. The lesson begins with students in small groups of two or three. In these groups, students will discuss and interpret what is happening in the Bathtub Graph. Their discussions should include explanations of differences or changes in the graph. Each group should write their explanations, which should include specifics about changes in the graphs and what could have caused those changes to occur. After small group discussions and written explanations, the teacher should facilitate a class discussion of differing interpretations, making the connection between the slope of the lines and what might have actually happened to produce the graph (see Teacher Notes Follow Up Discussion).
2. In the next activity, students will use the graphing calculators to investigate how a change in a linear equation will result in a change in the graph of the equation. The students will use a graphing calculator to complete the Investigating the Relationship between Linear Equations and their Graphs Worksheet (see Teacher Notes for this worksheet.). This activity is designed to be very open-ended so that the students will feel free to explore the relationship between a linear equation and its graph and then make their own conjectures.
3. For the third activity, students will work with a partner to identify slope as a rate of change, including appropriate units, from a linear graph on the Meaning of Slope: Rate of Change in Graphs Worksheet. The inclusion of units in the identification of the slope is central to this activity, because it gives the slope meaning in the context of the problem situation. When students have completed the worksheet, they should participate in a class

discussion, where they are required to explain the meanings of the slopes of the problems on the worksheet.

## ASSESSMENT PROCEDURES:

1. The teacher should monitor student discussions and student work on the Bathtub Problem by walking from group to group and listening in on the group discussions. The teacher should not direct the discussions, unless the students are misunderstanding and need some guidance to get on the right track. In this informal assessment, the teacher should determine if the students were correctly interpreting the graph and the changes in slope as they related to the situation that was represented by the graph.
2. The students should be able to express verbally and in writing their understanding of the connections between the graph, the changing slope, and the problem situation.
3. The students should be able to make a correct conjecture concerning the connection between the equation of a line and its slope.
4. As a follow-up assessment to the Investigating the Relationship Between Linear Equations and their Graphs Worksheet, the students could be asked to correctly identify slopes of lines represented by equations only.
5. The students should be able to identify the slope from a graph and include the appropriate units.
6. The students should be able to give the meaning of the slope as it applies to a particular problem situation.

## ACCOMMODATIONS/MODIFICATIONS:

Students who have trouble isolating the changes in the line on The Bathtub Graph should be allowed to use a ruler. By placing the ruler on one line segment at a time, the student can isolate individual changes in the graph.

Students can work in pairs on the graphing calculator to allow those who have fine motor difficulties to focus on investigating the relationship of the graph and the equation and not get bogged down in punching keys.

Students can also use uncooked spaghetti to lie across the screen of the graphing calculator to better visualize the graph of the line. The teacher can also use the uncooked spaghetti on the overhead graphing calculator screen to help the students visualize the graph.

## REPRODUCIBLE MATERIALS:

Bathtub Problem (used as overhead transparency or student worksheet)
Teacher Notes on Bathtub Problem for Follow-up Discussion (for teacher use only)
Teacher Notes on Investigating the Relationship Between Linear Equations and their Graphs (for teacher use only)
Investigating the Relationship Between Linear Equations and their Graphs Worksheet
Meaning of Slope: Rate of Change in Graphs Worksheet

## EXPLORATION AND EXTENSION:

Students could create their own sketches of graphs like the bathtub problem to model a particular situation. They should write the story first, then decide the quantities for the *x*- and the *y*-axis, and finally graph the situation.

Students can use the graphing calculator to investigate changes in the graph when the equation is not linear. For example, equations with $x^2$ or $x^3$ and absolute value equations.

## LESSON DEVELOPMENT RESOURCES:

McConnell, John W., et al. UCSMP Algebra Second Edition. Sunnyvale, CA; Scott Foresman, 1996.

## REFLECTIONS:

Students often know the formulas for finding slope, but have no real understanding of the importance of this quantity in a given situation. With the Bathtub Problem, students are unencumbered by numerical quantities on the graph. They are free to compare the various slopes without worrying about exact answers. Students seem to enjoy the opportunity to be creative in their explanations of this graph as they attempt to connect the graph to the situation. By having the students write up their explanations, the activity forces the students to think through and evaluate their own interpretations. By having the students share their explanations with the class, the students will begin to accept that there are different ways to correctly interpret the problem.

The open-ended investigation, in which the students use the graphing calculator to make connections between the graph of a line and its equation, allows students the freedom to make their own discoveries. They seem to remember and understand concepts that they figure out for themselves much better than when the teacher tells them the concept. Students seem to always be more interested in lessons that involve technology, in this case, the graphing calculator.

The first two activities in this lesson require the teacher to give the students time and freedom, two things essential to the success of this unit. The teacher must step back and take on the role of monitor in these activities. If the students are allowed ample time to investigate and feel free to investigate the concepts, then they will feel more responsible for their learning. They will have ownership in what they are learning.

For the Meaning of Slope worksheet, students are required to take what they know about slope and use it to give meaning to a particular situation. It is often difficult to get students to attach units to their solutions. Science teachers seem to have a similar problem. This activity requires students to include the units for the purpose of giving meaning to the slope. Relating the placement of units to the formula for slope $\left(\frac{\textit{vertical change}}{\textit{horizontal change}}\right)$ helps the students place the correct units in the numerator and in the denominator. (The units from the *y*-axis go in the numerator and the units from the *x*-axis go in the denominator.

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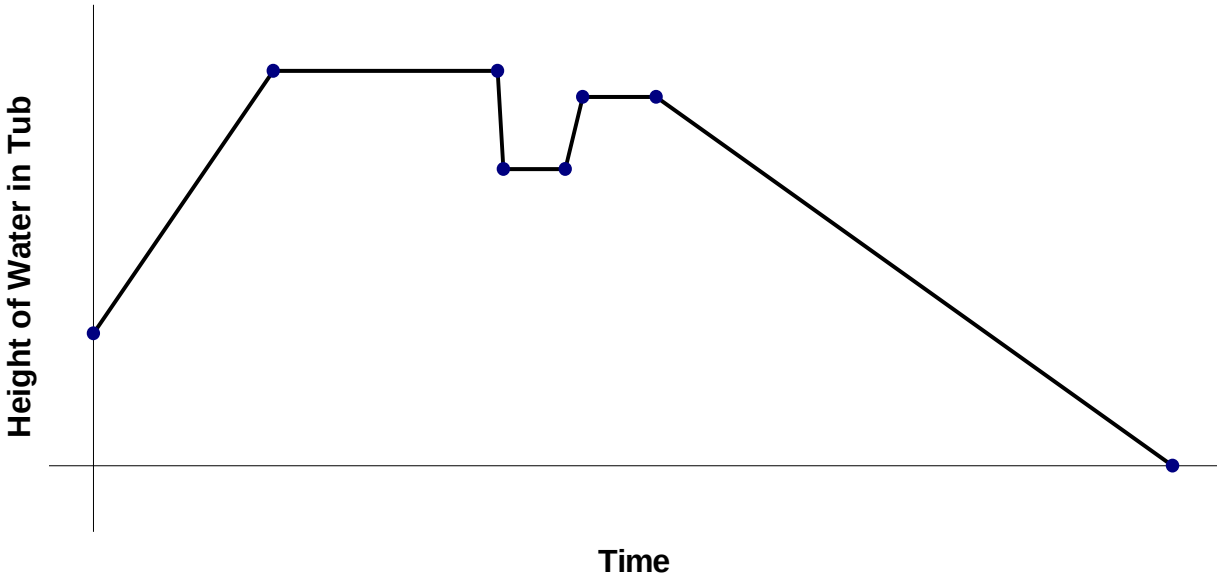
# BATHTUB PROBLEM




Write a story that can be described by the graph above. Explain changes in the graph by indicating what is happening to the water in the bathtub and what situation might have caused that to happen. Use the points labeled above when writing your story.

# Bathtub Problem
## Teacher Notes

### Follow-up Discussion

The following questions should be addressed in the discussion as students share their stories of the graph:

- Does the story start off with an empty tub?
  (There is already water in the tub as indicated by point A, the y intercept.)
- What is the meaning of the horizontal lines?
  (The height of the water in the tub is not changing.)
- Why do some lines go up from left to right and some lines go down?
  (Lines that go up indicate that the water level in the tub is rising. Lines that go down indicate that the water level is falling.)
- Is the line from point C to point D vertical? What might have caused it to be nearly vertical in this situation? Would it be possible for the line to be vertical? That is, could the level of water in the bathtub change without any time passing?
  (The line from C to D cannot be vertical, as that would indicate that the water level in the tub changed without any time passing.)
- Is there water left in the tub or does it all drain out?
  (There is no water left in the tub as indicated by the *x*-intercept, point H.)
- What rate of change does the slope of the line indicate?
  (The slope of the lines indicates the rate of the change of the height of water in the tub over time.)
- What is the explanation for the different slopes of the lines?
  (A line that is less steep indicates a slower rate of change than a line that is steeper.)

# Investigating the Relationship Between Linear Equations and their Graphs
# Using a Graphing Calculator
## Teacher Notes

**Materials Needed:** The teacher should have an overhead graphing calculator. Each student should have use of a graphing calculator. Two students can share one calculator if necessary.

**Overview:**

For this activity students should be introduced to inputting linear equations and viewing their graphs using the graphing calculator. Students will then be asked to investigate how changes in the equation affect changes in the graph of that equation. Students will be asked to make some conjectures concerning the relationship of a linear equation to its graph.

**Procedures:**

1. Using the overhead graphing calculator, the teacher should show students how to input an equation into a graphing calculator and then view the graph. Students might need some help in setting the proper window.
2. Working in groups of two's or three's, students should be asked to complete Investigating the Relationship Between Linear Equations and their Graphs: Using a Graphing Calculator Worksheet. They start by inputting a linear equation into the calculator and examine the graph. Students should be instructed to investigate how changing the linear equation results in changes in the line. The group should keep documentation of what changes in the equations produced what changes in the graphs.
3. After the investigation is complete, groups should present their conjectures to the class and discuss. The students should discover how changing the coefficient of *x* changes the steepness of the line, including making the line steeper, less steep, or horizontal. The students should also discover how to manipulate the coefficient of x so that the line can be made to go up from left to right or to go down from left to right. Students should make other discoveries, including how the *y* -intercept appears in a linear equation. This investigation will help students develop their own understanding of the Slope Intercept Form of a linear equation, $y = mx + b$, where $m$ is the slope and $b$ is the *y*-intercept.

**Extensions:**

- Use of the Table button to obtain the coordinates of two points in order to determine the actual value of the slope using two points on the line and the formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Use of the Calc button to get the graphing calculator to calculate the slope of the line.

**Investigating the Relationship Between Linear Equations and their Graphs**
**Student Worksheet**

1. Enter a linear equation into your graphing calculator.

2. Observe the graph of the equation on the calculator. Make sure that you have an appropriate window with which to view the graph.

3. Record your original linear equation and sketch the graph below.

4. Modify one number in your original equation and enter it into your graphing calculator.

5. Observe the graph and record the differences in appearance from the graph of your original equation.

6. Repeat steps 4 & 5 several times. Fill in the table below with each modified equation.

7. Make one or more conjectures based upon your findings. At least one of the conjectures should state some relationship concerning the slope of a line as it relates to the equations.

**Original Equation:**

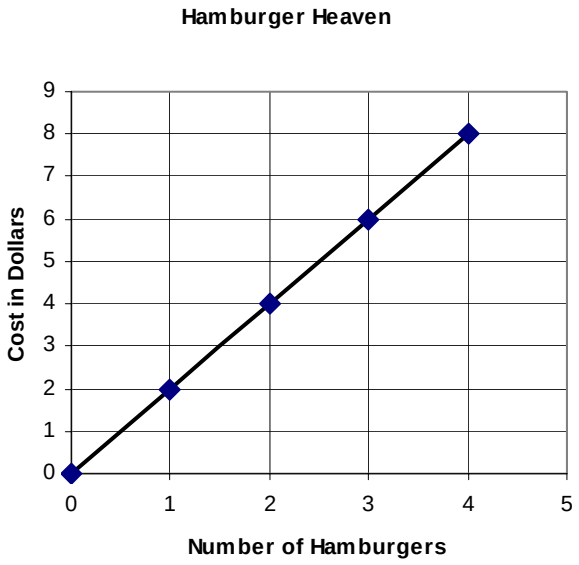
***y* =** _____ **Sketch :**

| **Modified Equation** | **Description of Differences from graph of original** |
|---|---|
| | |
| | |
| | |
| | |
| | |

**Conjectures:**

**Meaning of Slope**
**Rate of Change in Graphs**
**Student Worksheet**

**1)**



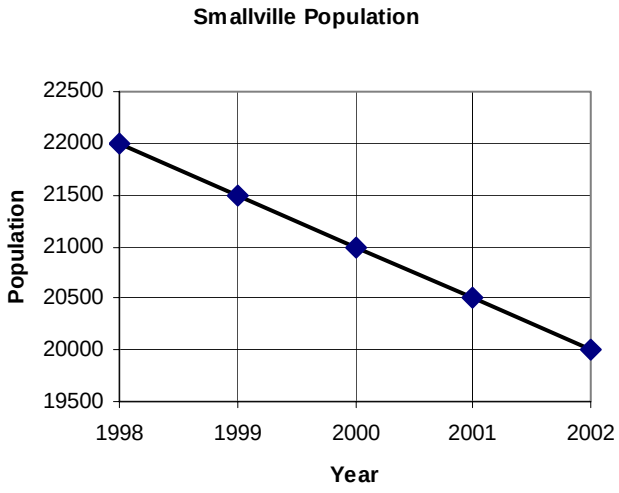

a) What is indicated by the fact that the line is going up from left to right?

b) According to the graph, what is the slope (rate of change) of the line? Be sure to include your units.

c) What is the meaning of the slope in this situation?

d) Should the points in this example be connected by a line? Explain.

**2)**




a) What is the unit length on the *x*-axis? What is the unit length on the *y*-axis?

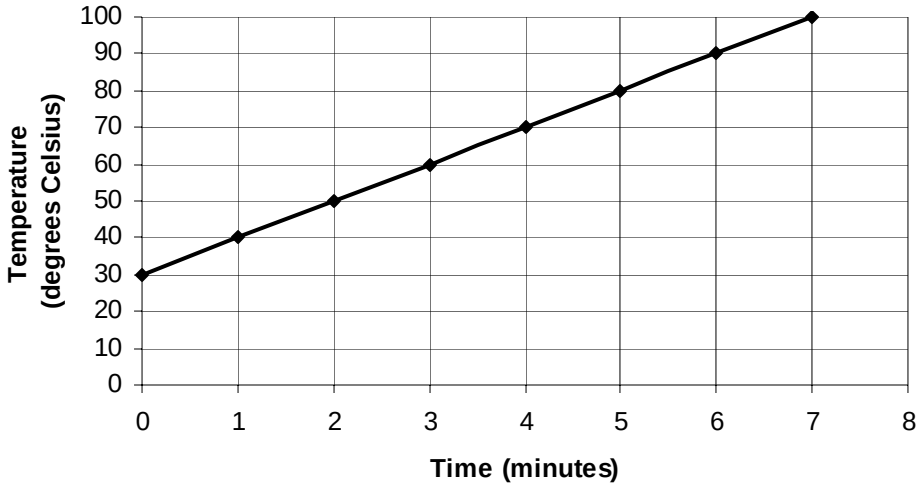
b) What is the slope (rate of change) of the graphed line? Be sure to include units.

c) What does the fact that the line is going down indicate? How is this shown in the slope?

d) What is the meaning of the slope in this situation?

**3)**

**Heating Water to a Boil**



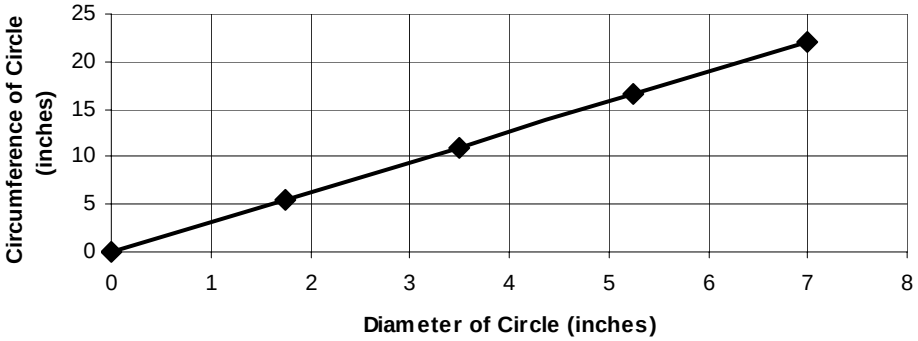

a) What is the slope (rate of change) for this graph?

b) What is the meaning of the slope in this situation?

c) What is the *y*-intercept and what is its meaning in this situation?

**4)**

**Comparing Circumference and Diameter**




a) What is the slope (rate of change) for this graph?

b) What is the meaning of the slope in this situation?

c) Identify the *y*-intercept and give it's meaning.

**Meaning of Slope**
**Rate of Change in Graphs**
**ANSWERS**

**1)**

**a) As the number of hamburgers increase, the cost in dollars also increases.**

**b) The slope can be determined by $\frac{rise}{run} = \frac{\$2}{1\text{hamburger}}$. The slope is $2 per hamburger**

**c) In this situation, the slope indicates the cost per hamburger. Each hamburger costs $2.**

**d) Since there is no meaning for the cost of a fraction of a hamburger (for example, ½ hamburger costs $1), the line is meaningless. The points should not be connected by a line.**

**2)**

**a) The unit length on the *x*-axis is 1 year. The unit length on the *y*-axis is 500 people.**

**b) The slope is –500 people per year.**

**c) Since the line is going down from left to right, this indicates that the population is decreasing during the time period 1998-2002. The negative sign in the slope illustrates this.**

**d) Smallville's population is decreasing by 500 people per year during the years 1998 through 2002.**

**3)**

**a) The slope is 10° C per minute.**

**b) The meaning of the slope is that the temperature of the water is rising 10°C for every minute that passes.**

**c) The *y*-intercept is 30° C. This is the temperature at zero minutes, when the process started.**

**4)**

**a) The slope of the line is $\frac{22}{7} \approx$ 3.14 inches in circumference per inch in diameter.**

**b) The slope is approximately equal to $\pi$ (pi), which is the ratio of the circumference to the diameter of a circle.**

**c) The y intercept is equal to 0. It means that a circle with a diameter of 0 inches has a circumference of 0 inches.**