Teacher-to-Teacher
Video Series
for Secondary Educators

TITLE: Polynomial Multiplication

PRIMARY SUBJECT AREAS: Algebra - Polynomials

GRADE LEVELS: 8-10

OVERVIEW: This lesson will emphasize using the distributive process to multiply polynomials. The lesson begins with an activity that connects area models and the distributive process to polynomial multiplication. The next activity uses graphing calculators as a method of investigating polynomial multiplication as it relates to graphing. The lesson will conclude with a problem-solving application of polynomial multiplication.

APPROXIMATE DURATION: 2-3 fifty minute class periods

LOUISIANA CONTENT STANDARDS:
http://www/DOE/assessment/standards/MATH.pdf

Algebra

A-1-H Demonstrating the ability to translate real-world situations into algebraic expressions, equations or inequalities and vice versa.

A-2-H Recognizing the relationship between operations involving real numbers and operations involving algebraic expressions.

A-3-H Using tables and graphs as tools to interpret algebraic expressions, equations, and inequalities.

GLEs Addressed

Grade 9

8. Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
9. Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
15. Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)

EDUCATIONAL TECHNOLOGY GUIDELINES:

http://www/DOE/LCET/curric/k12stand.pdf

Technology Productivity Tools

- Students use technology tools to enhance learning, increase productivity, and promote creativity.

INTERDISCIPLINARY CONNECTIONS:

N/A

OBJECTIVES:

1. The student will use geometric area to model the process of polynomial multiplication, and connect that multiplication to the distributive process.
2. The student will use graphing calculator technology to explore the relationship between polynomial factors, products, and their graphs.
3. The student will use polynomial multiplication to problem-solve.

LESSON MATERIALS AND RESOURCES:

Polynomial Multiplication: Area Models Worksheet
Polynomial Multiplication: Relating Factors, Products, and Graphs Worksheet
Polynomial Multiplication: Problem-Solving Worksheet
Worksheet Answer Keys (for teacher use only)
Algebra textbook
Algebra Tiles (may be used instead of diagrams)

TECHNOLOGY TOOLS AND MATERIALS:

Overhead graphing calculator
Graphing calculator for each student

BACKGROUND INFORMATION:

Students should have a basic understanding of the distributive process and of monomial multiplication. This lesson is best done with students who have experience with algebra tiles, but it can be modified for use with inexperienced students (see Accommodations/Modifications). Students should be able to determine the area of rectangles and squares. Students should know the difference between linear dimensions, such as width and length, and a two-dimensional area. Students need basic knowledge of the graphing functions of a graphing calculator, including entering an equation, graphing the equation, and adjusting the window.

LESSON PROCEDURES:

1. The teacher should review the concept of area of rectangles and squares.
2. Using Algebra tiles, the teacher should review the difference between the dimensions of length and width and the area. For this work, we will let the large square have a length of "x" and an area of "x^2". The rectangle will have a length of "x" and a width of "1", with an area of "$1x$" or "x". The smaller square will have a length of "1" and an area of "1".
3. After the above review, the students should work in groups of 2 to complete the Area Models Worksheet. The teacher may want to have a

class discussion after the first problem to make sure everyone is on the right track. A class discussion should again be held after everyone completes numbers 2 – 5. Students can then go on to work problem 6, with a discussion of the activity as a wrap-up. During the post-activity discussion, students should recognize that problems 6e and 6f would be difficult to do with algebra tiles, but they can easily be done with the distributive process.

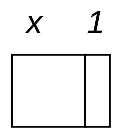
4. Each student will need a graphing calculator for the Relating Factors, Products, and Graphs Worksheet. The teacher should allow the students to work in groups of 2 or 3 to complete questions 1 – 7. Several students should be allowed to use the overhead graphing calculator to show the class how they worked through the questions. Students should be required to complete the problems at the bottom of the worksheet on their own. The wrap-up discussion should not include giving the answers since this is a method of checking their own work.
5. The students should approach the Problem Solving Worksheet in groups of two. After an initial discussion time between partners of 5 – 10 minutes, each student should be required to write up their advice. The rubric for this activity should be discussed and each student should have access to a copy. After the teacher has scored each solution and the students receive feedback, they should be required to complete and turn in a final draft of the problem solving activity.

ASSESSMENT PROCEDURES:

1. The teacher should monitor student discussions and student work on the Area Models Worksheet to assess whether the students are understanding the connection between the area models, the polynomial factors, and their products
2. On the Area Models Worksheet, each student should be able to find the correct product of the polynomial multiplication problems listed as number 6.
3. The students should be able to take a polynomial product and create a model for that product. The student should then be able to identify the dimensions of the model.
4. The teacher should informally assess student understanding of the Relating Factors, Products, and Graphs Worksheet through class or group discussions.
5. The students should be able to correctly multiply the polynomial problems at the bottom of the Relating Factors, Products, and Graphs Worksheet. The students should be able to determine the correctness of their answers using the graphing calculator.
6. The students should be able to write a convincing argument about their solution to the Problem-Solving Worksheet. The argument should be supported with drawings or models, algebraic equations, and numerical representations. (See rubric.)

ACCOMMODATIONS/MODIFICATIONS:

For the Area Models Worksheet, students with no experience using algebra tiles, for whatever reason, can be accommodated by having the teacher write in the dimensions for each of the smaller parts of the larger rectangles for every problem. For example



Students who are tactile learners should be asked to model the figures on the Area Models worksheet using actual algebra tiles.

REPRODUCIBLE MATERIALS:

Polynomial Multiplication: Area Models Worksheet
Polynomial Multiplication: Relating Factors, Products, and Graphs Worksheet
Polynomial Multiplication: Problem-Solving Worksheet

EXPLORATION AND EXTENSION:

Students can create area models using algebra tiles to obtain the factors (length and width) from the product (area). Students would be given a polynomial product (area), such as $x^2 + 4x + 3$, and asked to model a rectangle with that area using algebra tiles. Students would then be asked to give the length and width of that area and connect those dimensions to the factors of the product (area).

Students can discuss how the knowledge of the *x*-intercepts of a polynomial equation will be helpful in solving the equation. Students can discuss other non-mathematical factors that might enter into the decision-making process used in the Problem Solving Worksheet. Students can explore through examples and discussion how that same problem might be easier or more difficult to solve if the plot of land was a rectangle.

LESSON DEVELOPMENT RESOURCES:

McConnell, John W., et al. UCSMP Algebra Second Edition. Sunnyvale, CA; Scott Foresman, 1996.

Explorations: Algebra with the TI-83 Plus. Videotape. Texas Instruments Incorporated, 2001.

REFLECTIONS:

Using the geometric area models to investigate polynomial multiplication allows students to connect a physical model with an algebraic concept. Often it is difficult for many of my students to understand polynomial multiplication. By connecting the factors to the width and the length of the model and by connecting the product to the sum of the individual areas of the model, students, especially visual learners, gain a better understanding of the distributive process. In the past I have emphasized the FOIL process. Students were so busy concentrating on following the steps (first, outside, inside, last) that they failed to gain a good understanding of the distributive process.

In using the graphing calculator as a method of checking polynomial multiplication, students begin to make a connection between the graph and the polynomial. Students enjoy using the graphing calculators because by checking their own work they do not have to rely on the teacher to determine their understanding. They can assess their own understanding. The Relating Factors, Products, and Graphs worksheet is laying the groundwork for solving quadratic equations by factoring and by finding the zeros.
The Problem-Solving worksheet is an opportunity to bring writing into my classroom. So often my students want to put down a numerical solution but cannot explain why their answer is the best choice. In this writing exercise the students are required to back up their answer with justification in the form of models, graphs, charts, examples, and algebra.

I find it valuable to require students to show their work and then explain it. By putting their answers into words, they are required to think about the problem on a deeper level. The answer alone will not be enough, they must explain.

Students are so often under the misconception that there is only one right way to work a math problem. This lesson allows them to work through math problems using geometric models, graphing calculators, numerical examples, charts and graphs, and algebra. They seem to enjoy the variety.

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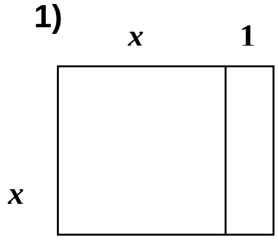
Polynomial Multiplication
Area Models Worksheet

1)



a) Using the area of the individual parts of the rectangle (the large square and the small rectangle), indicate what the total area would be.

__________ + ____________
(square) (rectangle)

b) What polynomial represents the length of the rectangle?______________

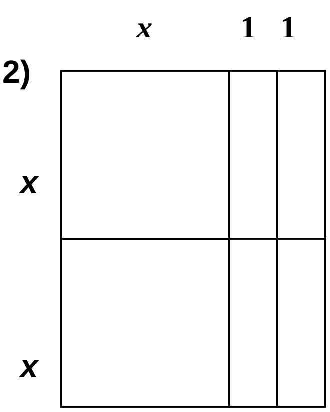
What polynomial represents the width of the rectangle?______________

c) Area of a rectangle is defined by the equation *length* • *width* = *area*. Use your answers to the above questions to fill in the area equation below.

_________________ • _________________ = __________________________
length *width* *total area*

d) What is the relationship between the factors and the product? What property is illustrated?

2)



a) Express the area of the rectangle as a sum of the individual areas.

____________ + ______________________

b) Give the expression for the length of the rectangle_______________

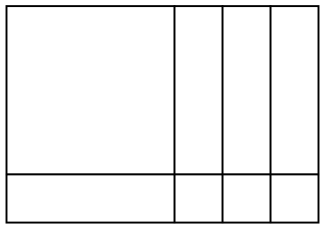
Give the expression for the width of the rectangle________________

c) Give the equation for the area of the rectangle using the area, length, and width above. Be sure to express the area as a polynomial in simple form.

_________________ • _________________ = __________________________

d) Is the distributive property illustrated in the above equation? Explain.

3) *Use the same dimensions for the large and small squares and the small rectangles for the remainder of the problems*.



a) Express the area of the polynomial as a sum of the individual areas that make up the larger rectangle.

__________ + _________ + ___________

b) Give the polynomial that represents the length. _____

Give the polynomial that represents the width. _____

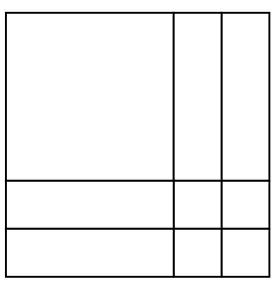
c) Give the equation for the area of the rectangle.

________________ • ________________ = ______________________

d) How could the entire area be determined without using the areas of the individual pieces?

e) How would the distributive property be used differently in this problem than in the previous ones?

4)



a) Give the polynomial that represents the length. _________

Give the polynomial that represents the width. _________

b) What do you notice about the polynomial for the width and the polynomial for the length?

c) What does this tell you about the rectangle?

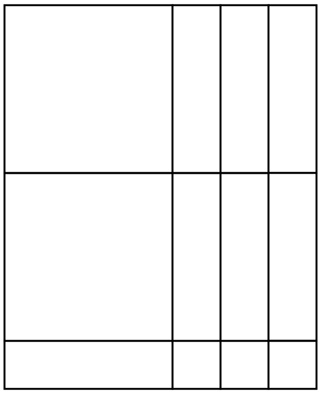
d) Give two different equations for the area.

(1) ________________ = _______________ (2) __________________ = _______________

e) Explain how the factors and the product of the equation relate to the individual pieces of the square.

f) Since this is an area problem, how should the units be represented?

5)



a) Express the length as a polynomial. ___________

Express the width as a polynomial. ___________

b) Give the equation for the area of the rectangle. Be sure to express the area as a polynomial in simple form.

_____________ • _____________ = ______________

c) Did you use individual areas or distributing to determine the area?

d) Are the length and the width interchangeable as factors? Does it matter which way is the length and which way is the width?

What property of real numbers does this illustrate?

Does this property apply to polynomial multiplication also? Give an example.

6) Determine the products of the following. Use the Distributive Property (and area models if needed).

a) $2x(x + 4) =$

b) $(x + 5)(x + 2) =$

c) $(x + 2)(2x + 1) =$

d) $(x + 8)(x + 4) =$

e) $(2x + 9)(3x + 1) =$

f) $(3x - 2)(x + 6) =$

Polynomial Multiplication
Relating Factors, Products, and Graphs Worksheet

This worksheet will help us use graphing calculators to check our polynomial multiplication.

1) Give the product of $(x + 2)(x - 3)$ = ________________

2) Enter the factors as a *"y ="* equation in your graphing calculator.

$$y = (x + 2)(x - 3)$$

3) Observe the graph of the equation. You might have to adjust your window to see the graph on the calculator.

4) Enter the product as a *"y ="* equation in your graphing calculator. Enter this equation in "y_2" and be sure that the equation, $y_1 = (x + 2)(x - 3)$, is still visible.

y_2 = ________________
(product from step 1)

5) If the factors and the product are equal, what should the graph of the product look like? Observe the graph.

6) What can you conclude if the graph of the equation of the factors is not the same as the graph of the equation of the product?

7) How many times does the graph cross the x-axis? At what points? What are the y- coordinates of these points?

PROBLEMS: Simplify the following. Use your graphing calculator to check your products by graphing the equation for the factors and the equation for the product. Also indicate where the graphs cross the x-axis.

a) $(x + 5)(x - 2)$ = ____________________________ x-intercepts __________

b) $(x + 7)(x + 3)$ = ____________________________ x-intercepts __________

c) $(x + 3)(x - 3)$ = ____________________________ x-intercepts __________

d) $(2x + 1)(3x - 4)$ = __________________________ x-intercepts __________

e) $(x + 5)(x + 5)$ = ___________________________ x-intercepts __________

Polynomial Multiplication
Problem Solving Worksheet

Your family owns a square plot of land on the edge of town, and they look to you to advise them on how to manage it. A real estate agent offers you a deal. He wants to trade your square plot of land for a rectangular plot that is 10 feet longer in length and 10 feet shorter in width. Is this a fair trade? Would your family gain land? What would you advise your family to do about the offer? Your family usually does not listen to your advice unless you back it up with evidence. Use numbers, models, and algebra to support your advice.

Write a well-developed paragraph explaining what advice you would give your family and why.

Polynomial Multiplication
Problem Solving
<u>RUBRIC</u>

Understanding Concepts	-translated the problem into appropriate mathematical concepts, including algebra - used more than two problem solving strategies (models, diagrams, charts, graphs, computations, tables, algebraic sentences)
Knowledge	-demonstrated and applied appropriate mathematical procedures, computations, and formulas -analyzed mathematics and proposed a meaningful solution to the problem
Interpretation	-showed strong evidence of reasoning in analysis and interpretation -referenced their mathematical work in their writing -included alternative interpretations when appropriate
Communication	-used good sentence structure and proper grammar -precisely used effective mathematical terminology -identified all important elements of the problem situation -used appropriate and clearly labeled mathematical models

Polynomial Multiplication
Area Models Worksheet
ANSWERS

1) a) x^2 + 1x
(square) (rectangle)
b) x and $x + 1$ Length and width are interchangeable.
c) $x \bullet (x + 1) = x^2 + x$
length *width* *total area*
d) When the length is multiplied by the width, the resulting product is the sum of the areas. This is an example of the distributive property.

2) a) $2x^2 + 4x$
b) $2x$ and $x + 2$ Length and width are interchangeable.
c) $2x \bullet (x + 2) = 2x^2 + 4x$
d) Yes, the distributive property is illustrated by multiplying $2x$ by both the x and the 2.

3) a) $x^2 + 4x + 3$
b) $x + 1$ *and* $x + 3$ Length and width are interchangeable.
c) $(x + 1) \bullet (x + 3) = x^2 + 4x + 3$
d) The area could be determined by multiplying, using the distributive property, the length and the width. $(x + 1)(x + 3) = x^2 + 3x + 1x + 3$
$= x^2 + 4x + 3$
e) In this problem, more than one term is being distributed. Each term in the first parentheses (both the x and the 1) must be multiplied by each term in the second parentheses (both the x and the 3).

4) a) $x + 2$ and $x + 2$ Length and width are interchangeable.
b) Both the length and the width are the same.
c) The rectangle is a square.
d) (1) $(x + 2)(x + 2) = x^2 + 4x + 4$ (2) $(x + 2)^2 = x^2 + 4x + 4$
e) The factors are the sum of the lengths of the individual pieces along each side of the large area. The product is the sum of the individual areas.
f) All units for area are square units.

5) a) $2x + 1$ and $x + 3$ Length and width are interchangeable.
b) $(2x + 1) \bullet (x + 3) = 2x^2 + 7x + 3$
c) Answers will vary. Get students to explain an example of each type.
d) Length and width are interchangeable as factors. The side that is considered the length can also be called the width since the rectangle can be rotated 90° and still have the same dimensions and area.
Commutative Property of Multiplication: length • width = width • length
Yes. $(2x + 1) \bullet (x + 3) = (x + 3) \bullet (2x + 1) = 2x^2 + 7x + 3$

6) (a) $2x^2+8x$ (b) $x^2+7x+10$ (c) $2x^2+5x+2$ (d) $x^2+12x+32$ (e) $6x^2+29x+9$
(f) $3x^2+16x-12$

Polynomial Multiplication
Relating Factors, Products, and Graphs Worksheet
ANSWERS

1) $(x + 2)(x - 3) =$ $\mathbf{x^2 - x - 6}$

3) **The graph should be a parabola with a vertex (minimum point) of (0.5,-6.25)**

4) $y_2 =$ $\mathbf{x^2 - x - 6}$
(product from step 1)

5) **If the factors and the product are equal, the two equations should produce identical graphs.**

6) **A mistake has been made in multiplying the factors. The product is incorrect.**

7) **The graph crosses the *x*-axis twice at -2 and 3. The *y*-coordinates at these two points are zero.**

PROBLEMS:

a) $(x + 5)(x - 2) =$ $\mathbf{x^2 + 3x - 10}$ — x-intercepts **-5 and 2**

b) $(x + 7)(x + 3) =$ $\mathbf{x^2 + 10x + 21}$ — x-intercepts -**7 and -3**

c) $(x + 3)(x - 3) =$ $\mathbf{x^2 - 9}$ — x-intercepts -**3 and 3**

d) $(2x + 1)(3x - 4) =$ $\mathbf{6x^2 - 5x - 4}$ — x-intercepts $-\frac{1}{2}$ **and** $\frac{4}{3}$ **or** $1\frac{1}{3}$

e) $(x + 5)(x + 5) =$ $\mathbf{x^2 + 10x + 25}$ — x-intercepts **-5**

Polynomial Multiplication
Problem Solving Worksheet
ANSWERS

Your family owns a square plot of land on the edge of town, and they look to you to advise them on how to manage it. A real estate agent offers you a deal. He wants to trade your square plot of land for a rectangular plot that is 10 feet longer in length and 10 feet shorter in width. Is this a fair trade? Would your family gain land? What would you advise your family to do about the offer? Your family usually does not listen to your advice unless you back it up with evidence. Use numbers, models, and algebra to support your advice.

Write a well-developed paragraph explaining what advice you would give your family and why.

Students should write a well developed paragraph that fulfills all the conditions of the rubric.

They could include a chart with several numerical examples

	Length	Width	Area
Original Plot	20 yards	20 yards	400 square yards
Real Estate Offer	10 yards	30 yards	300 square yards

From a chart such as this, the student should conclude and state that the real estate offer is a bad deal and the family would lose 100 square yards of property in the deal.

They could use algebra to compare the original plot of land to the real estate deal offered.

Original Plot of Land **area =** x^2 **(since the plot is square)**
Real Estate Offer **area =** $(x - 10)(x + 10)$
$= x^2 - 100$

They could draw a diagram to illustrate that the deal would cost the family 100 square yards.